GRESHAM COLLEGE

The Maths of Coins and Currencies

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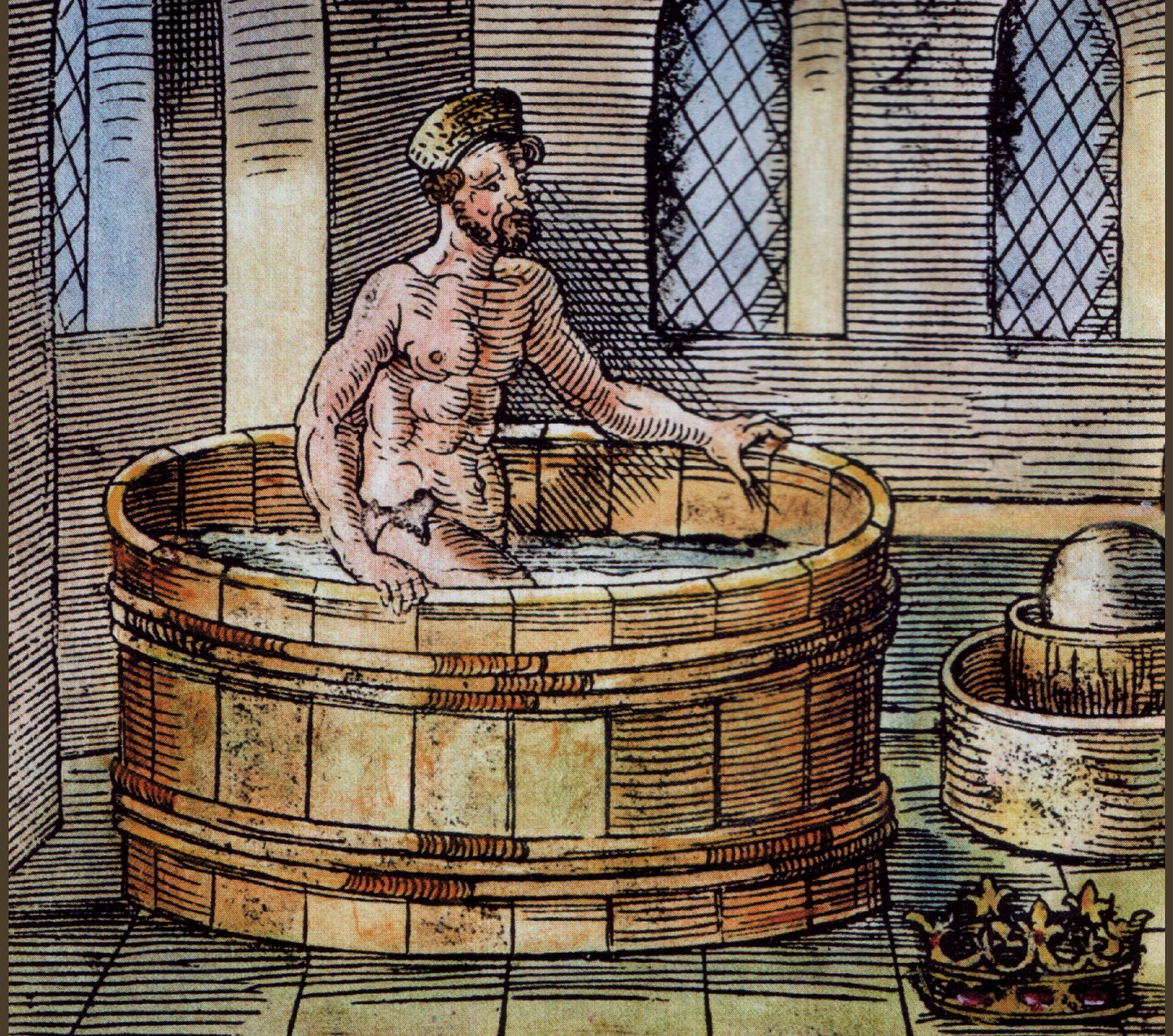
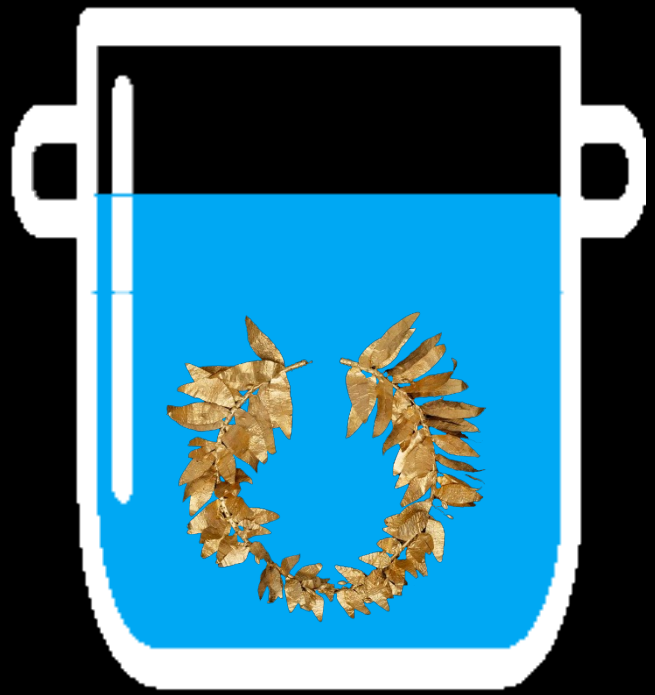
Hiero’s Crown



Was Vitruvius wrong?

- density of gold: 19.32g/cm^3; of silver: 10.49g/cm^3
- density $= \frac{\text{mass}}{\text{volume}}$, so volume $= \frac{\text{mass}}{\text{density}}$,
- 1kg gold is $\frac{1000}{19.32} = 51.76$ cm^3;
- 1kg silver = 95.33 cm^3
- if 1kg crown actually 10% silver, then
  900g gold (46.58 cm^3) + 100g silver (9.53 cm^3)
- total 56.12 cm^3, a discrepancy of 4.36 cm^3.



- diameter 20cm, area ≈ 314 cm^2
- volume difference 4.36cm^3
- difference in water level 0.14mm.

- tube 1cm wide
- height in tube is 4.36 cm.

Galileo’s suggestion

- Use Archimedes’ knowledge of buoyancy.
- Weigh equal amounts gold & silver
- 1kg gold = 51.76cm^3
- Apparent weight underwater 948.24g
- 1kg silver = 95.33cm^3
- Apparent weight underwater 904.67g

Which weighs more –
a ton of bricks or a ton of feathers?

Catching counterfeits

- Weighing
- Measuring
- Assaying
- Milling etc

How to run a mint

- Suppose you have two kinds of silver bullion:
  - Bad bullion: fineness 4 (ounces in a pound say)
  - Good bullion: fineness 9
- Coins must have fineness 7. What do we do?

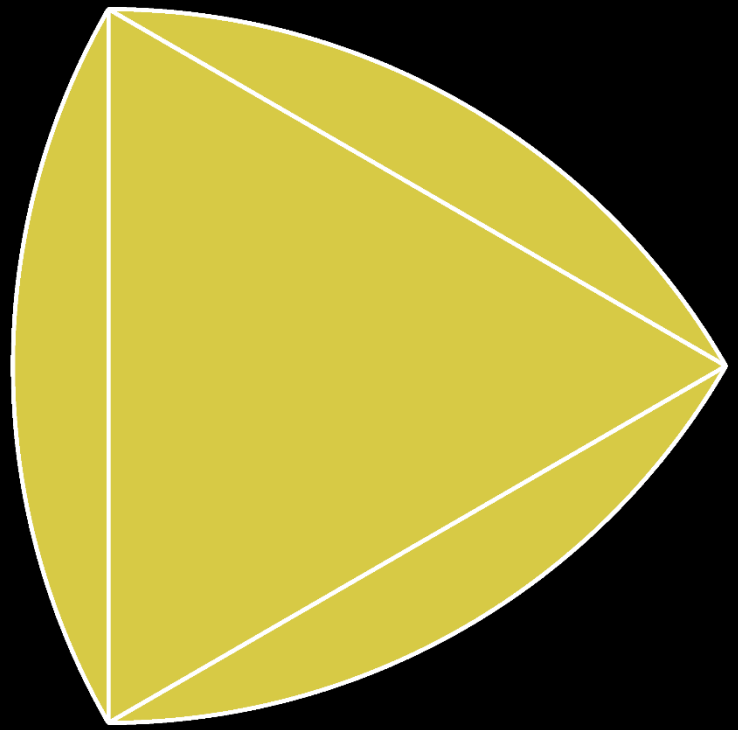
Fibonacci's alligation solution:

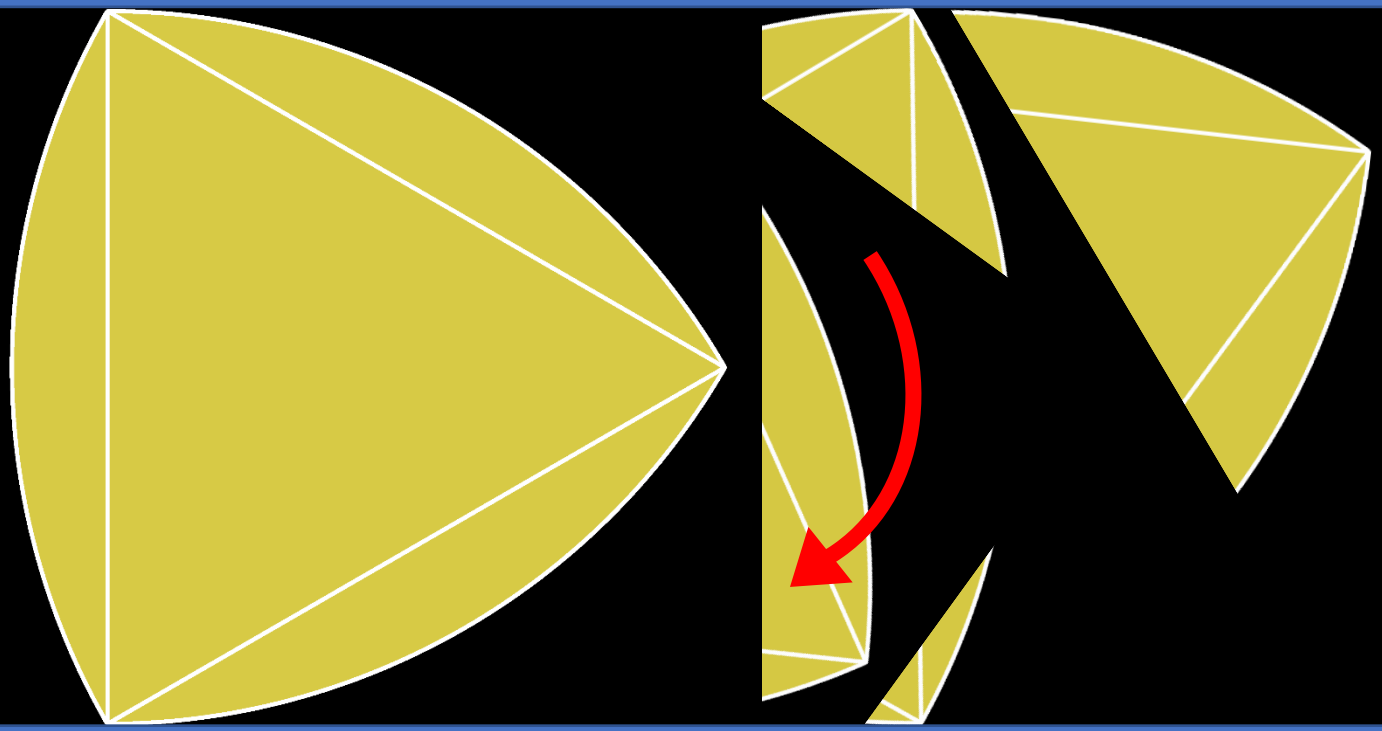
- Bad bullion is 3 parts worse than needed
- good bullion is 2 parts better.
- "Therefore" mix in ratio 2 bad: 3 good
- Can generalise. Many applications.

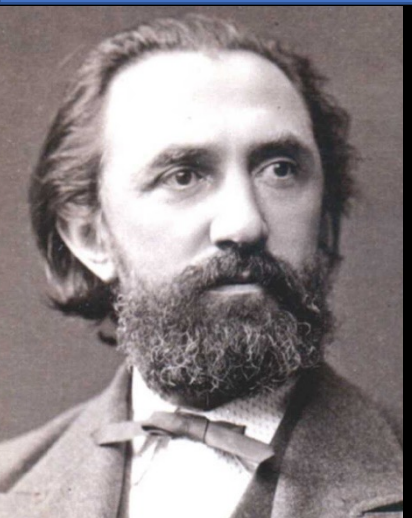
The puzzle of the birds

- 30 birds of 3 kinds bought for 30 pennies
- sparrow = ½p, pigeon = 2p, partridge = 3p
- desired “fineness” therefore 1 (penny/bird)
- sparrow : pigeon = 1 : ½ or 2 : 1 (total 3)
- sparrow : partridge = 2 : ½ or 4 : 1 (total 5)
- *Any combination* of these will give 1p/bird
- Only one solution involves all three birds:
- 3 partridge 5 pigeon 22 sparrows
- Subject now known as linear programming

The shape of money







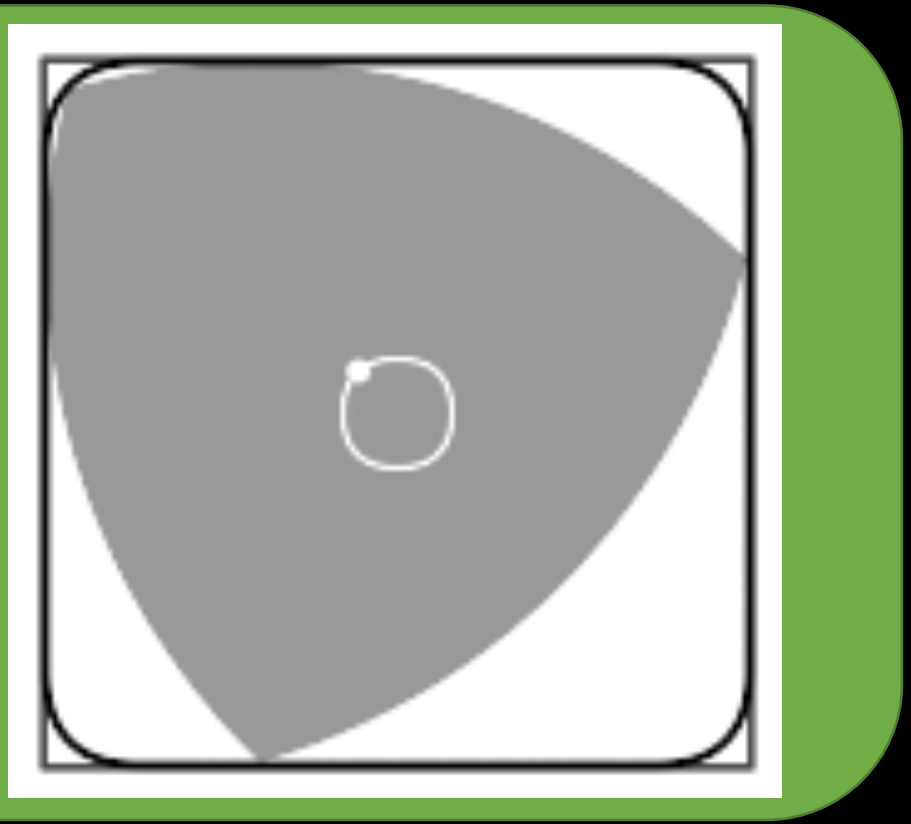
- Side length of triangle is s; each circle arc radius s
- Constant width s
- Works for any odd number of sides: Reuleaux polygons

Reuleaux coins

Other uses of Reuleaux triangles

Guitar pick/plectrum

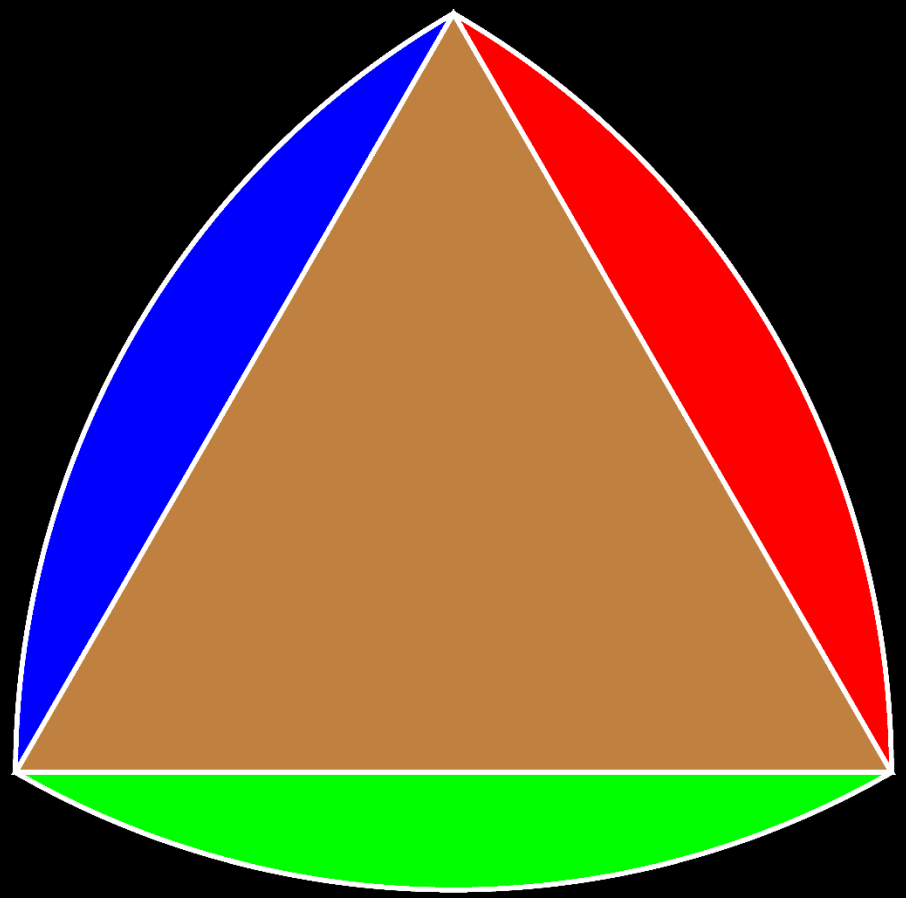
- points for sharp, bright sound;
- curves for soft, warm sound



Drilling a square hole

- Harry Watts drill bit (1916)
- Image credit: Wikipedia

Area of shapes of constant width



- Barbier's theorem: every shape of constant width s has perimeter πs.
- Area Reuleaux Triangle width s
- $= 3\left(\frac{1}{6}\text{circle}\right) - 2(\text{triangle})$
- $= \frac{1}{2}(\pi s^2) - 2\left(\frac{1}{2} \cdot s \cdot \frac{\sqrt{3}}{2}s\right)$
- $= \frac{1}{2}\pi s^2 - \frac{\sqrt{3}}{2}s^2$
- $= \frac{1}{2}s^2\left(\pi - \sqrt{3}\right) \approx 0.705s^2$
- circle width s has area $\frac{1}{4}\pi s^2 \approx 0.785s^2$
- Reuleaux Triangle least possible

straight edges, min/max
width difference too large

Curved edges, min/max
width difference smaller

The denomination debate

- Want: easy to divide up into parts, and easy to calculate with.
- To make $^1/_2$, $^1/_3$, $^1/_4$, need units divisible into a multiple of 12 parts.
- 1 shilling = 12d

Why go decimal?

- Old £1 = 20s = 240d.

- Cost of 40 items at 1s 7d each:
- 280d = 23s 8d = £1 3s 8d
- 40s = £2
- Total: £3 3s 8d

- If £1 = 100 pennies then cost of 40 items 8p each is £3.20.

Worst currency ever?

= 17

= 29

- 1696: Christopher Wren proposed silver noble divided into 10 primes, 100 seconds

  "which Centessimal division will be very proper for Accounts"

- 1704: Russian rouble = 100 kopeks
- 1792: US dollar = 100 cents
- UK finally went decimal 15 Feb 1971

What (decimal) denominations are best?

Dr Adam Townsend (University of Durham) suggested average number of coins in change as measure of efficiency.

- UK: 1p, 2p, 5p, 10p, 20p, 50p
- Up to 99p change:
- Mean 3.43 coins, median 3
- Swap 20p for “crown”?
- No effect

- US: 1¢, 5¢, 10¢, 25¢
- Up to 99¢ change:
- Mean 4.75, median 5
- Swap quarter for 20¢?
- Mean 5.05

Try it: my version of code, + links to Adam Townsend’s articles, in transcript

- self-checkout machines usually give 1p, 2p, 5p, 20p, £1, £2
- change up to 99p:
  mean 4.75, median 5

- with 1p, 2p, 5p, 20p, 50p, £2
- change up to 99p:
  mean 3.84, median 4

Has the 1p had its day?

- ½p coin "*let not its existence be imperilled. It is indispensable for levelling off pendulum clocks*" discontinued 1984.
- £1 in 1984 worth £2.63 today*
- ½p → 1.3p

- £1 in 1960 worth £16.18 today*
- 1 farthing → 1.7p

*BoE inflation calculator goes to end 2021

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The Mathematics of Game Theory

22nd November 2022, 1pm

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