Option Pricing
Professor Raghavendra Rau
26 February 2024

Introduction

My name is Raghavendra Rau and I'm a professor at the University of Cambridge. This is the third in a series of Gresham lectures this year on the big ideas of finance. The lectures this year are drawn from my textbook A short introduction to corporate finance, published by Cambridge University Press.

As I said before, there are only six major ideas in finance, five of which have won their originators Nobel prizes in economics. What are these ideas?

1. **Net Present Value (NPV):** NPV is the key principle in investment decision-making, where the objective is to maximize the present value of future payoffs. It involves three steps: computing cash flows, discounting those flows to a single present value using a discount rate, and deciding the financing method, which affects taxes.

2. **Portfolio Theory and the Capital Asset Pricing Model (CAPM)**

- The interest rate or discount rate in NPV is determined by investors, based on available investment opportunities. Markowitz and Sharpe, Nobel laureates, proposed that individual investments are parts of portfolios. They combined portfolios with risk-free assets to determine the market portfolio. The discount rate is determined using the CAPM formula.

3. **Capital Structure Theory:**

- Capital structure explains how the discount rate changes based on the firm's financing decision – whether to go with debt or equity. Modigliani and Miller, Nobel winners, posited that in a perfect world, financing form doesn't affect firm value. But with real-world imperfections (like taxes), it does matter.

4. **Option Pricing Theory:**

- This is the idea we are discussing in this lecture. It draws on Chapter 5 of my book. It discusses how to price options, which are contracts that give rights to buy or sell assets. Black, Scholes, and Merton, with the latter two winning a Nobel, provided a solution based on the no-free lunch principle. They matched the cost of a portfolio replicating an option's payoff to the option's cost.

5. **Asymmetric Information:**

- Lecture 5 deals with information imbalances in transactions, where one party has more information than the other. Akerlof, Spence, and Stiglitz, Nobel laureates, developed key concepts in this area, illustrating how information imbalances affect markets from used cars to financial policies.

6. **Market Efficiency:**

- The last lecture discusses how markets reflect all available information. The debate lies in the relationship between market prices and NPV. Three Nobel winners, Kahneman, Fama, and Shiller, contributed pioneering ideas on this topic, discussing market behavior and efficiency.

In essence, corporate finance revolves around six central ideas, with five of them recognized by Nobel Prizes.

What are options?

In the first lecture in this series, we discussed what finance was all about. At its core, finance is about promises. Someone promises you a fantastically large amount of money in return for some money today. The task for every financial decision is to understand whether these promises are worth the initial investment. The first three lectures covered the different ways we valued these promises (by calculating cash flows – lecture 1; by calculating the discount rate – lecture 2; and by estimating how much that discount rate changes with the level of debt we take on - lecture 3.

In this lecture, we examine *contingent* investments. Can you change your mind after making the investment? What is the value of being able to change your mind?

To answer that question, we need to turn to the fourth big idea in corporate finance – option pricing theory. Myron Scholes and Robert Merton won the Nobel Prize in Economics in 1997 for coming up with this idea. Like all the ideas so far, the intuition is deceptively simple.

An option is essentially a contract that offers the buyer the right, but not the obligation, to buy or sell an asset at a predetermined price within a specified timeframe. To understand this concept, let's consider a relatable, everyday example. Imagine you're planning to attend the renowned Shakespeare Festival in Cambridge. Given the unpredictable English weather, you face a dilemma: purchasing tickets in advance might lead to disappointment if it rains, while waiting could mean missing out if the event sells out or ticket prices surge beyond your budget. Here, the weather acts as an uncertain element, much like the volatility seen in financial markets.

In financial terms, securing a “call option” on your ticket would be an ideal solution. By paying a small fee upfront, you gain the right to buy the ticket at today’s price if the weather is favorable, but you're not obligated to complete the purchase if it rains. This arrangement eliminates your risk of either missing the play due to sell-out or enduring it in poor weather, paralleling how investors use options to hedge against market uncertainties.

However, this flexibility comes at a cost. Just as the box office might charge a fee for the option to buy a ticket under certain conditions, in the financial markets, options come with a premium. This premium compensates the seller for the risk of price movements against their position. The process of determining this price involves understanding the underlying asset's value and its potential volatility—challenges that had puzzled economists and traders alike.

The groundbreaking work of Fischer Black, Myron Scholes, and Robert Merton introduced a theoretical framework to price options by considering the underlying asset's market dynamics without needing to predict specific outcomes. Their model leverages the concept of constructing a risk-free portfolio, mirroring the payoff of the option, thereby allowing the market's collective wisdom to dictate the fair value of the option through the law of no arbitrage.

This approach has not only revolutionized the way financial markets operate but also underscored the intricate relationship between risk, time, and value—a fundamental principle that extends beyond finance to everyday decisions, like whether to attend an open-air Shakespeare play in the unpredictable English summer.

Forwards and futures vs. options

To understand how options might be priced, let’s start with a simpler concept – a forward or a future - and compare these two securities to options.

Futures are financial contracts *obligating* the buyer to purchase an asset or the seller to sell an asset at a predetermined future date and price. There is no choice here. The asset involved could be anything - a physical commodity or a financial instrument. Futures and forwards are similar - in both cases, you agree on a price and a future execution date for a trade. The key difference is that futures contracts are standardized and traded on exchanges.

Let’s use a simple analogy to explain futures, drawing from the scenario of buying a ticket for an outdoor Shakespeare play in Cambridge. Imagine you want to buy a ticket for a play that's happening a year from now. The spot price, or the price of the ticket if you were to buy it today, is £15. But you don't want to pay until the day of the show. The box office agrees to this but at a forward price, considering how the value of money changes over time. If the annual risk-free rate is 1%, the forward price you'd agree on might be

£15.15, taking into account the time value of money.

In futures trading, this concept is applied on a large scale. If the market thinks the weather will be bad, the price of your ticket might drop, and vice versa. Futures contracts can be bought and sold in the secondary market, allowing you to lock in prices or speculate on future price movements of various assets, from agricultural products to financial instruments.

The futures market includes mechanisms like marking to market, where gains and losses are tallied daily, and contracts are adjusted accordingly. This process ensures that the risk of default is minimized, as both parties must maintain a margin account to cover potential losses. Standardization and the marking-to-market mechanism make futures contracts less risky compared to forwards, as they provide a clear, regulated framework for trading.

Futures are used for hedging against price changes (like ensuring you don't pay more for your ticket if the weather is nice) or for speculative purposes (betting on price movements of commodities or financial assets). However, it's crucial to remember that while futures can protect against some risks, they can introduce new ones, such as the risk of losing money if market conditions move against your position.

How do we price options?

Options are a type of financial derivative that gives you the right, but not the obligation, to buy (call option) or sell (put option) an asset at a predetermined price (exercise price) on or before a specific date (expiration date). They can be used to hedge against potential losses or to speculate on the price movement of assets with a limited risk.

Let’s simplify the concept using a relatable example similar to buying a ticket for a future event. Suppose you’re interested in attending a Shakespeare play in Cambridge, and today's ticket price is £15. However, you’re unsure about the weather on the day of the play. To mitigate the risk of bad weather, you decide to buy an option to purchase the ticket at £15 (the exercise price) on the day of the show.

This option costs you a small fee upfront (the option premium), granting you the right to decide on the day of the show whether to buy the ticket at £15 or not. If the weather is favourable, you can exercise your option, buy the ticket at the pre-agreed price, and enjoy the play. If it rains, you can choose not to exercise the option, losing only the small fee you paid, rather than the full ticket price.

Options come in two main varieties: European, which can only be exercised on the expiration date, and American, which can be exercised at any time before the expiration date. The value of an option depends on several factors, including the current price of the underlying asset, the exercise price, the time until expiration, and the asset's volatility.

To understand how to price an option, we can first set up boundary conditions. The no-free lunch idea means that option prices cannot lie outside particular boundaries. For example, an option's price is never more than the asset itself. All option boundaries rest on the idea of no arbitrage opportunities. If the option price is outside this boundary, a free lunch is possible - where riskless profits can be made.

One key boundary in option pricing is put-call parity, which shows the relationship between the price of a call option and a put option with the same strike price and expiration date. This relationship helps ensure that the pricing of options remains consistent and prevents arbitrage.

The binomial model

The binomial model is a straightforward way to understand option pricing. It assumes that the price of the underlying asset (like our Shakespeare play ticket) can only go to one of two possible prices by the expiration date. For simplicity, say a ticket can either increase to £25 or decrease to £5 by show day.

Step 1: Understanding the Scenario

Imagine you’re considering buying a ticket for this play that's happening in the future. However, there are two main uncertainties: the weather (it could either be perfect or rain) and the play's popularity (it could be a hit or not attract much attention). These uncertainties closely parallel the two possible directions a stock price can take in the binomial model: up or down.

Step 2: The Decision Points

In the binomial model, we break down the time until the option expires into several steps, where at each step, the price of the underlying asset (in this case, the desirability of attending the play) can go up or down. Applying this to our scenario:

- **If the weather is forecasted to be great and the play becomes a must-see event**, the value of attending the play goes up. This is like the stock price rising in the binomial model.
- **If the forecast predicts rain and interest in the play wanes**, the value of your ticket or desire to attend drops, akin to the stock price falling.

Step 3: Pricing the Option (Decision to Buy the Ticket)

When you buy an option, you're essentially paying for the right to make a decision in the future (in this case, deciding later whether you want to attend the play). The binomial model helps us determine the price of this option (the right to decide) by considering all possible future scenarios (weather/popularity combinations) and working backward from the play's date to the present.

- **Creating a Riskless Portfolio**: Imagine you could create a "portfolio" by mixing your option to buy a ticket with some risk-free investment (like a government bond). By adjusting how much you invest in the bond versus the option, you aim to make your portfolio's return certain, no matter what happens with the weather or the play's popularity. This concept mirrors how the binomial model uses hedging to eliminate risk.
- **Risk-Neutral Valuation**: This approach assumes that all investors are indifferent to risk. Under this assumption, the option's price is determined by the expected value of the ticket's desirability, discounted back at a risk-free rate. It's like saying, “Given the uncertainties, what's the fair price today for the right to decide on attending the play later?”
- **Replicating the Option**: Lastly, the model suggests you could replicate the payoff of owning the option (the right to buy the ticket later) by dynamically adjusting your mix of the risk-free investment and direct investment into the play (buying the ticket outright now or deciding not to go). This replication strategy helps determine the fair price of the option without directly considering the specific risk preferences of any investor.

In essence, the binomial model breaks down the future into a series of binary (up or down) decisions, much like deciding on attending an outdoor play based on evolving factors. By considering all possible outcomes and their probabilities, then working backward from the future to today, the model helps calculate the fair price to pay for the flexibility to decide later. This approach gives us a straightforward, step-by-step way to understand the potentially complex pricing of options in the financial world, using the relatable scenario of planning for an outdoor event.

The Black-Scholes model

The Black-Scholes model builds on the principles of the binomial model but applies to a continuous setting, where stock prices can change in an infinite number of ways, not just two. The model assumes:

- Stock prices follow a random walk-in continuous time.
- No dividends are paid out during the option's lifespan.
- Markets are frictionless (no transaction costs or taxes, and securities are perfectly divisible).
- Investors can borrow and lend at a risk-free interest rate.

The Black-Scholes formula calculates the price of a European call option with the formula involving the current stock price, the strike price, the risk-free interest rate, the time to expiration, and the stock's volatility. The formula outputs the option's price by considering how the stock price's potential movements can affect the option's value at expiration.

Let's go back to buying a ticket for the outdoor Shakespeare play in Cambridge, where both the weather and the play's popularity can greatly influence our experience.

1. **Ticket Price Movements**: The fluctuating prices of play tickets as similar to stock prices. The Black-Scholes model assumes stock prices move in a continuous, somewhat predictable fashion. In our analogy, this is like trying to gauge how the demand for play tickets might change based on past trends, such as increased interest in Shakespeare plays during certain times of the year or

under specific weather conditions.

2. **Time**: Just as in options trading, time is a crucial factor in our decision to buy a play ticket. The closer we get to the play's date without buying a ticket, the less uncertain we become about whether we'll be able to attend (the weather is more predictable, we know our plans better). So the value of the option goes down – we might be happy to buy the ticket today instead of waiting to the final day (and paying a non-refundable premium for doing so). Similarly, an option's value is influenced by the time left until its expiration. The more time before the play, the more chances the weather forecast and popularity of the play can change, affecting your decision to buy a ticket now or later. This increases the value of the option.
3. **Strike Price vs. Current Price**: In options terms, the strike price is the pre-set price at which the option holder can buy or sell the underlying asset. In our analogy, consider the "strike price" as the ideal price you're willing to pay for a play ticket. Just as an option becomes more valuable when the stock price moves favourably relative to the strike price, your willingness to buy the ticket increases as the conditions (weather and play's appeal) align with your ideal scenario.
4. **Risk-Free Rate**: This factor is like choosing a risk-free, indoor event with a fixed enjoyment value instead of an outdoor play that depends on good weather. The risk-free rate in the financial world is what you'd earn from a completely safe investment, influencing the option's value by offering a baseline comparison. It's the guaranteed alternative you forgo when opting for the uncertain pleasure of the outdoor play.
5. **Volatility**: In our Shakespeare play scenario, volatility is represented by the unpredictability of the weather and the play's reception. High volatility in the stock market means the price could swing widely, similar to how sudden weather changes can significantly affect the desirability of attending an outdoor play. Just as higher volatility increases an option's value (due to the greater chance of the stock price hitting the strike price), the uncertain elements of weather and popularity make buying the play ticket a more thrilling (if riskier) bet.

By blending these elements—just as the Black-Scholes model does for options—we can evaluate whether buying a ticket for the outdoor play is a worthwhile gamble. We consider how much time is left until the play, how the play's popularity and the weather might change, and what we're giving up by not choosing a safer entertainment option. This approach helps us decide on the fair price to pay for the chance to enjoy Shakespeare under the stars, balancing our desire for a great cultural experience with the risks posed by English weather and the variable allure of the Bard's works.

All the lectures so far discuss the big ideas of finance when the market has perfect information. But this is rarely true. In reality, all transactions have information imbalances, where one party has more information than the other. We will discuss asymmetric information in lecture 5. Akerlof, Spence, and Stiglitz, Nobel laureates, developed key concepts in this area, illustrating how information imbalances affect markets from used cars to financial policies.

References and Further Reading

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637-654.

Rau, Raghavendra, “A short introduction to corporate finance”, 2017, Cambridge University Press (Chapter 4).