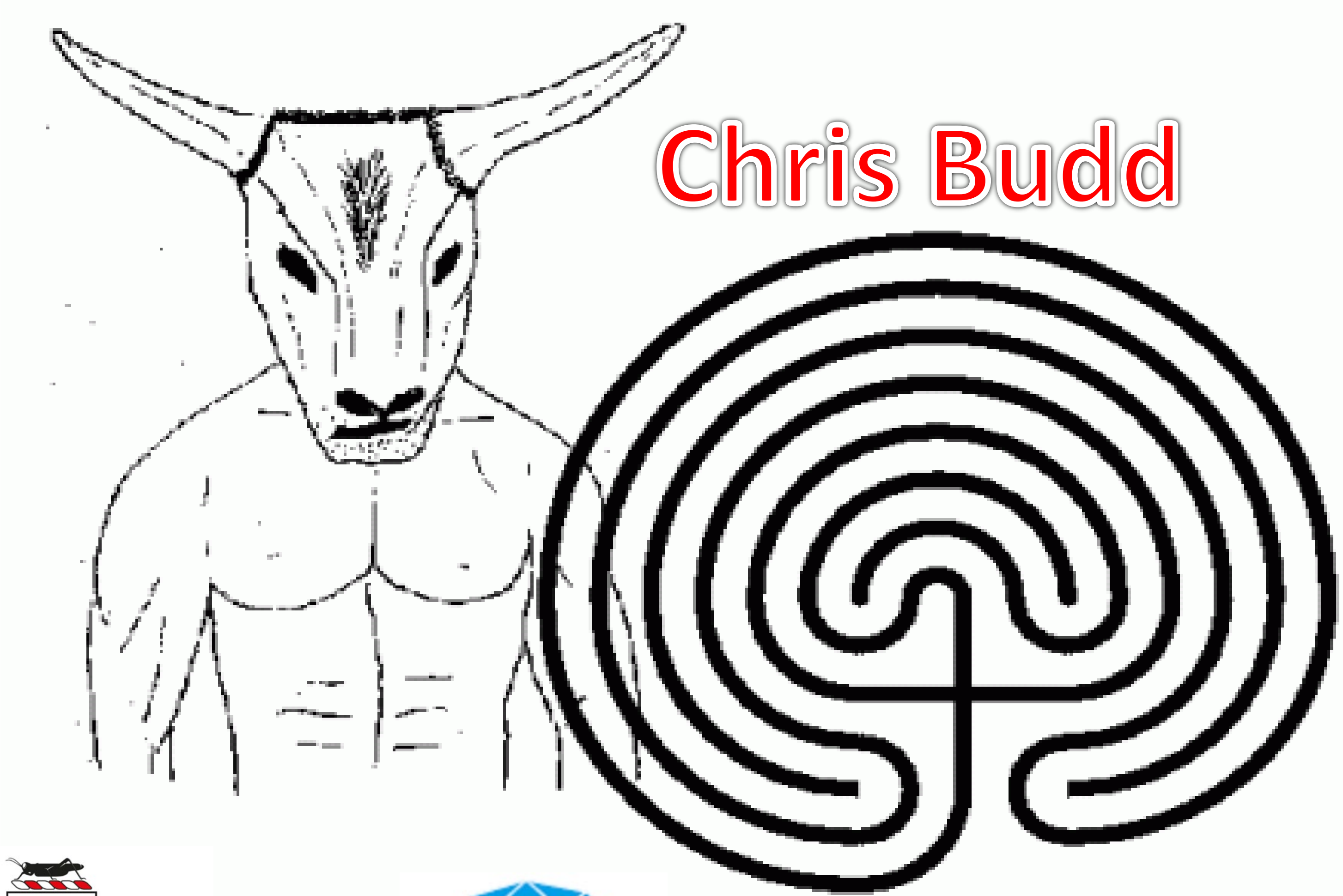
Great Mathematical Myths



Chris Budd

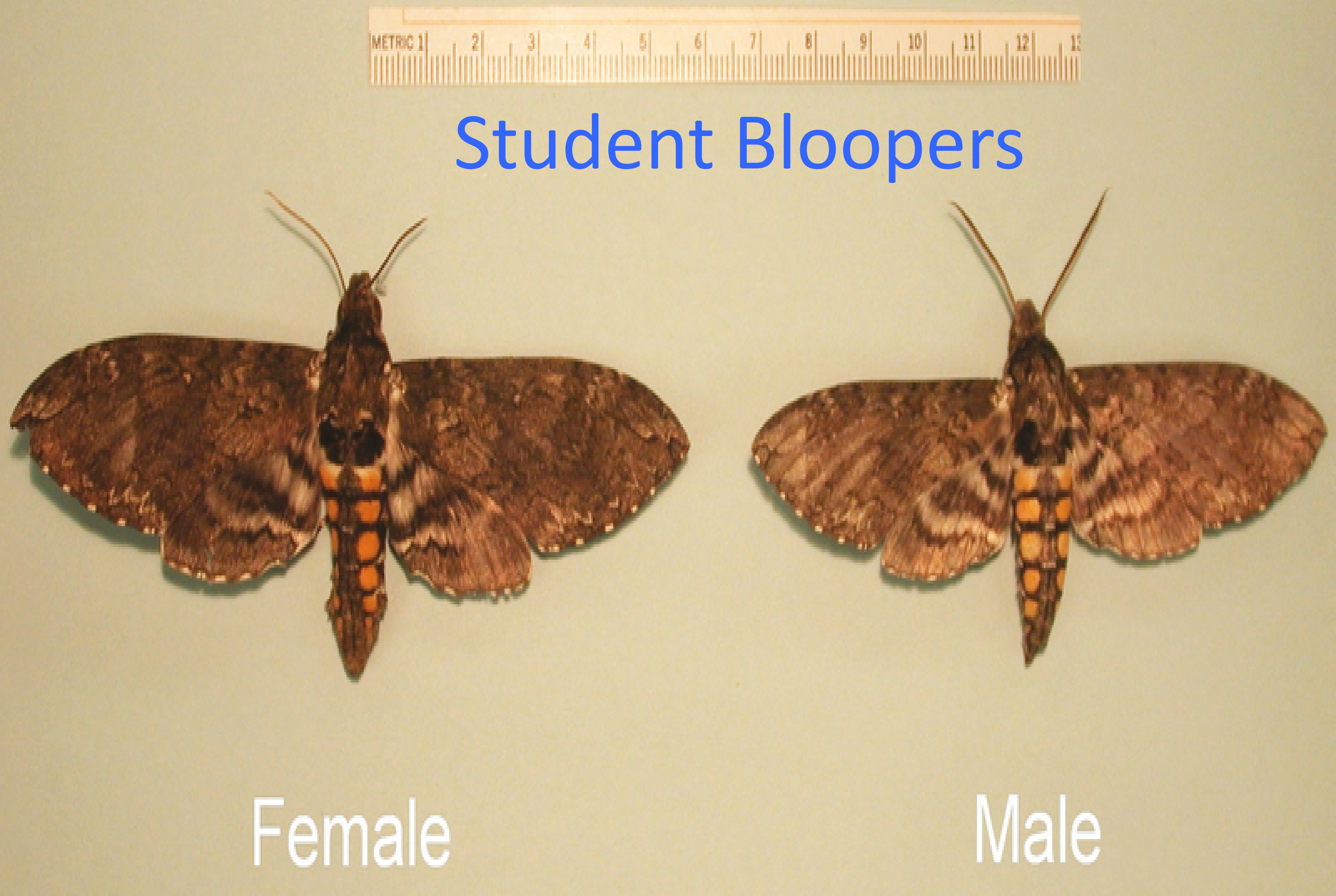
Gresham College

IMI

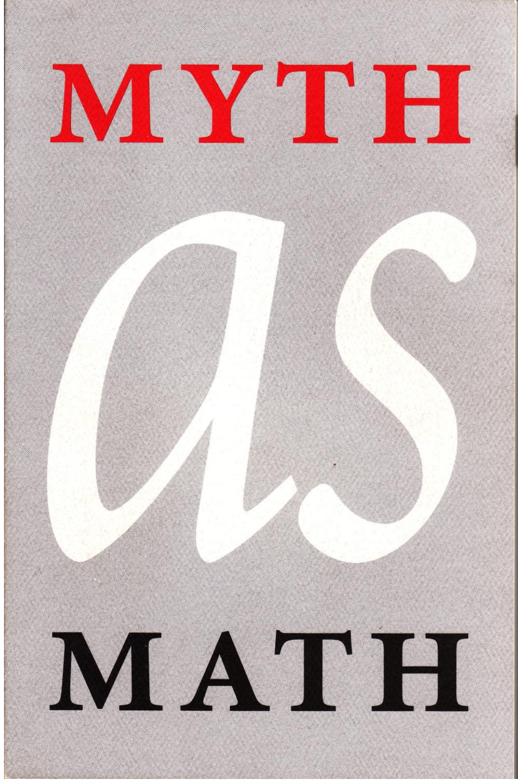
University of Bath

A myth is a female moth

Student Bloopers



When we think of Maths, we tend not to think of Myths



Myths are the stuff of legend and wonder

Maths on the other hand, is coldly logical and has no room for doubt and error!

Or does it?

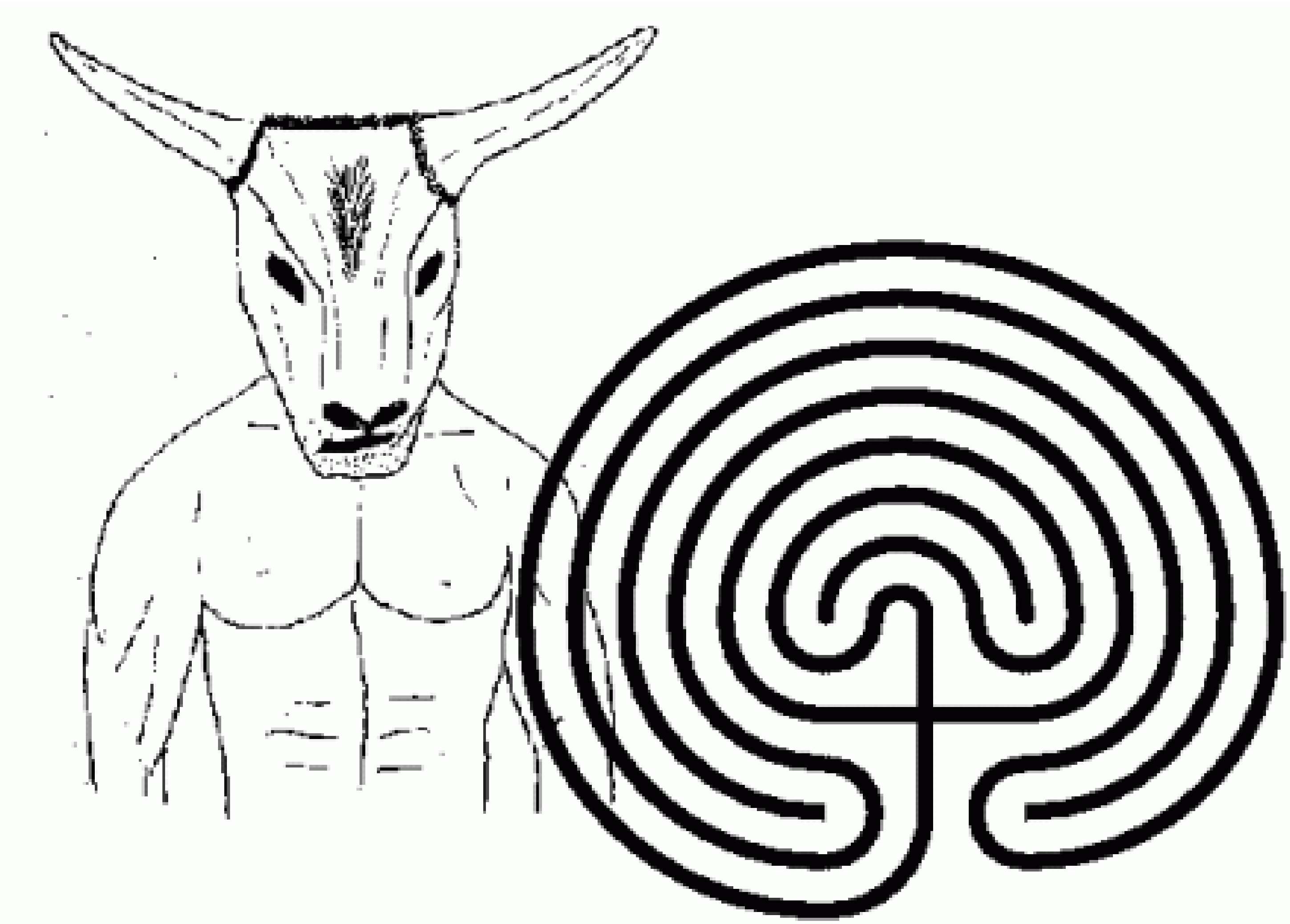
A mathematical truth, through retelling, an lack of understanding, enters the public consciousness as a myth rather than a truth

Especially the case if this satisfies some underlying need for some order and pattern in life, the universe, and everything

A great shame as mathematical truths are far more exciting and surprising than any myth, and give much insight into the way that the universe works

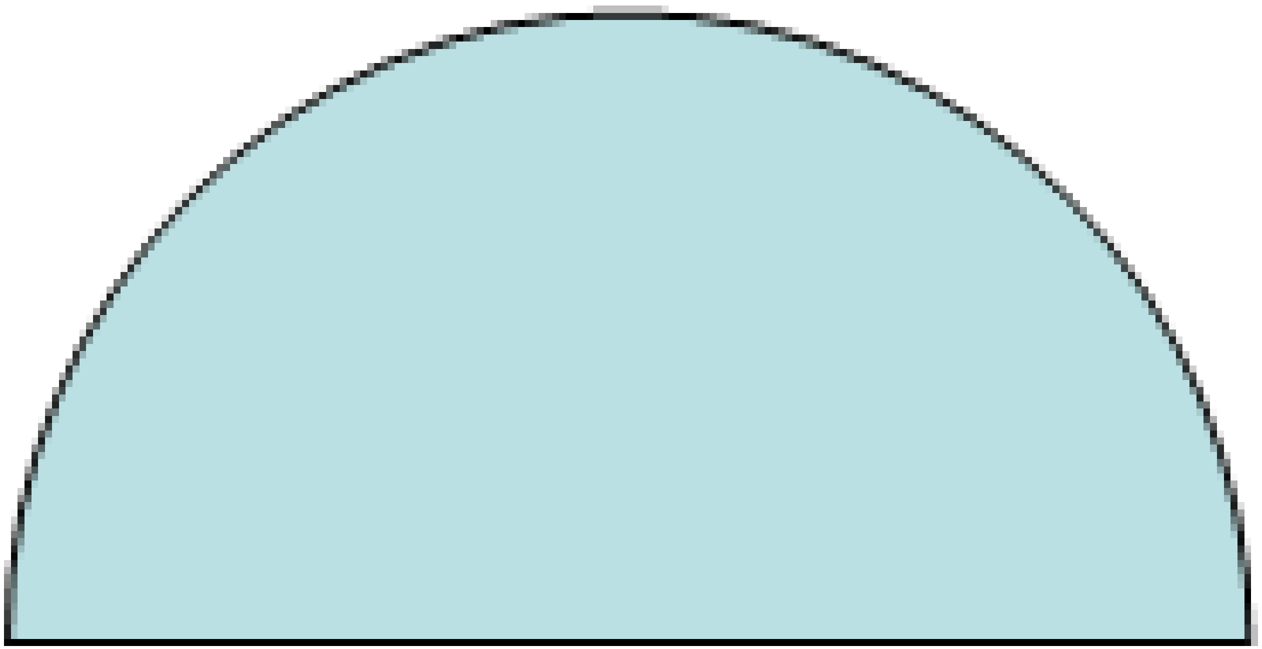
Myths about maths

Theseus and the Minotaur



Dido of Carthage

Dido Purchases Land for the Foundation of Carthage. Engraving by Matthäus Merian the Elder, in *Historische Chronica*, Frankfurt a.M., 1630. Dido's people cut the hide of an ox into thin strips and try to enclose a maximal domain.



Iso-perimetric theorem

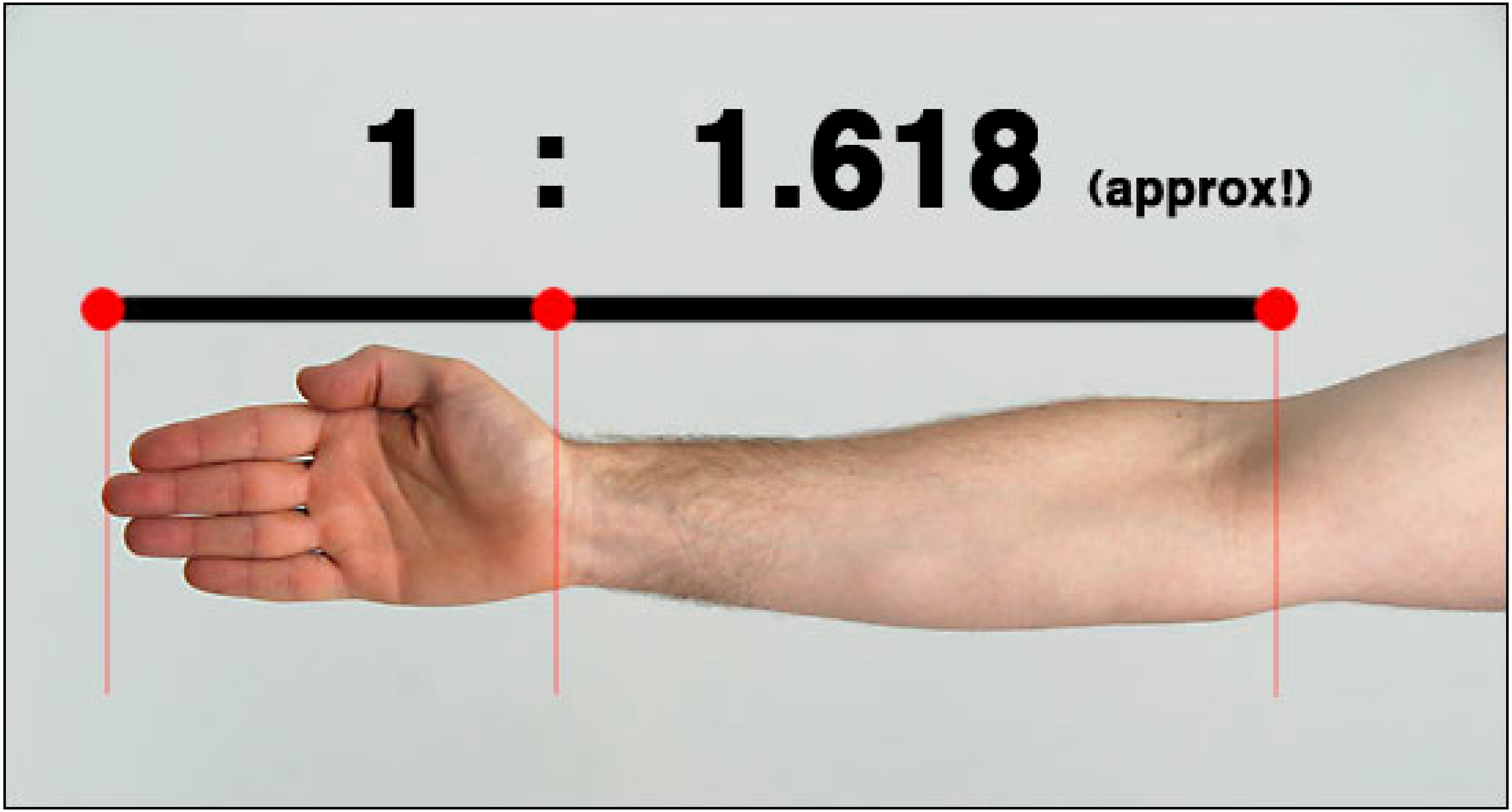
The scary
maths myth

Mathematical myths

The Golden Ratio

$$\phi = 1.61803\ldots$$

The claim …



The Golden Ratio is a Divine Proportion

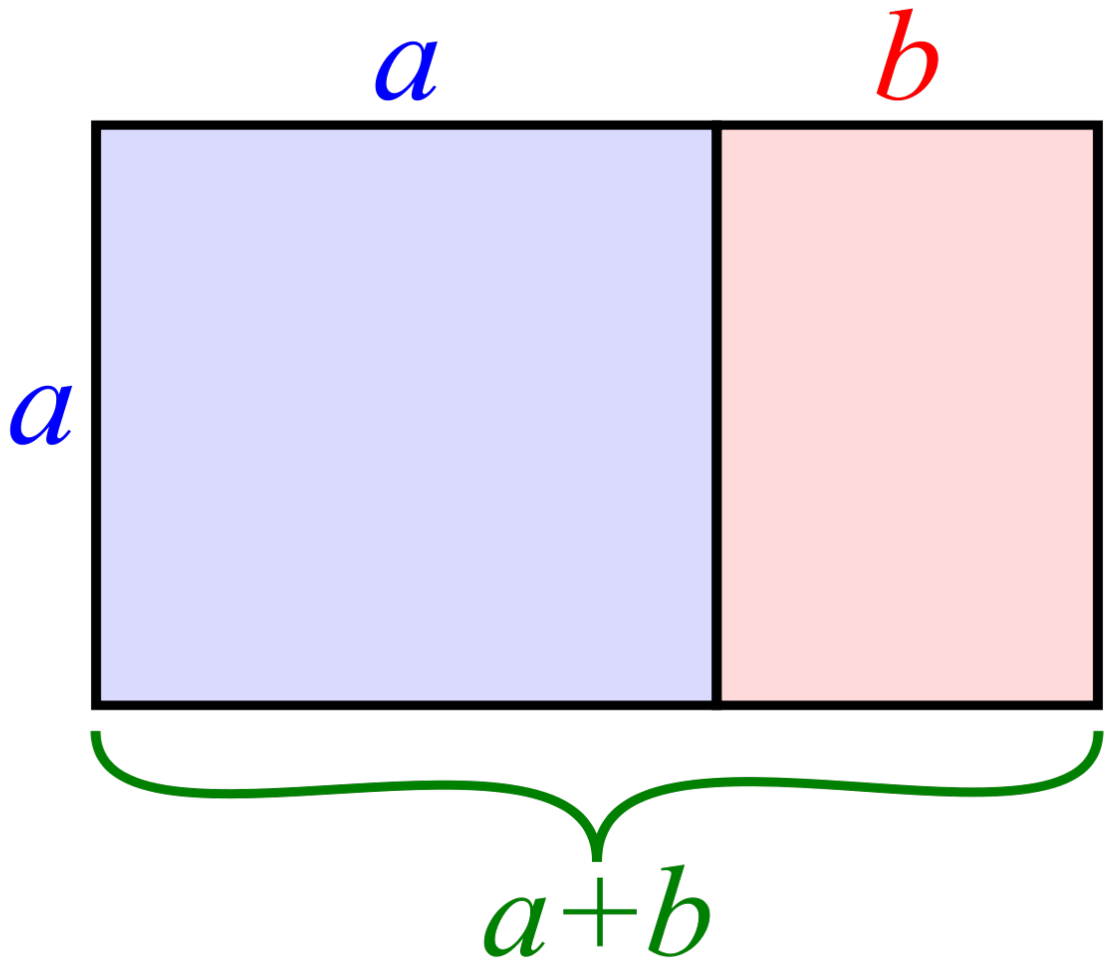
According to Mario Livio

Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece, through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler to present-day scientific figures such as Oxford physicist Roger Penrose have spent endless hours over this simple ratio and its properties. ... Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity and appeal. In fact, it is probably fair to say that the ***Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.***

The Truth about the Golden Ratio

$$\frac{a}{b} = \frac{a+b}{a}$$

Golden Rectangle



$$a/b = \phi > 1$$

$$\phi = 1 + \frac{1}{\phi}$$

$$\phi^2 = \phi + 1$$

$$x = \frac{1+\sqrt{5}}{2}, \quad y = \frac{1-\sqrt{5}}{2}$$

$$\phi = \frac{1 + \sqrt{5}}{2} = 1.61803\ldots$$

Link to the Fibonacci Sequence [Kepler]

1 1 2 3 5 8 13 21 34 55 89 144 …

Ratio between successive terms

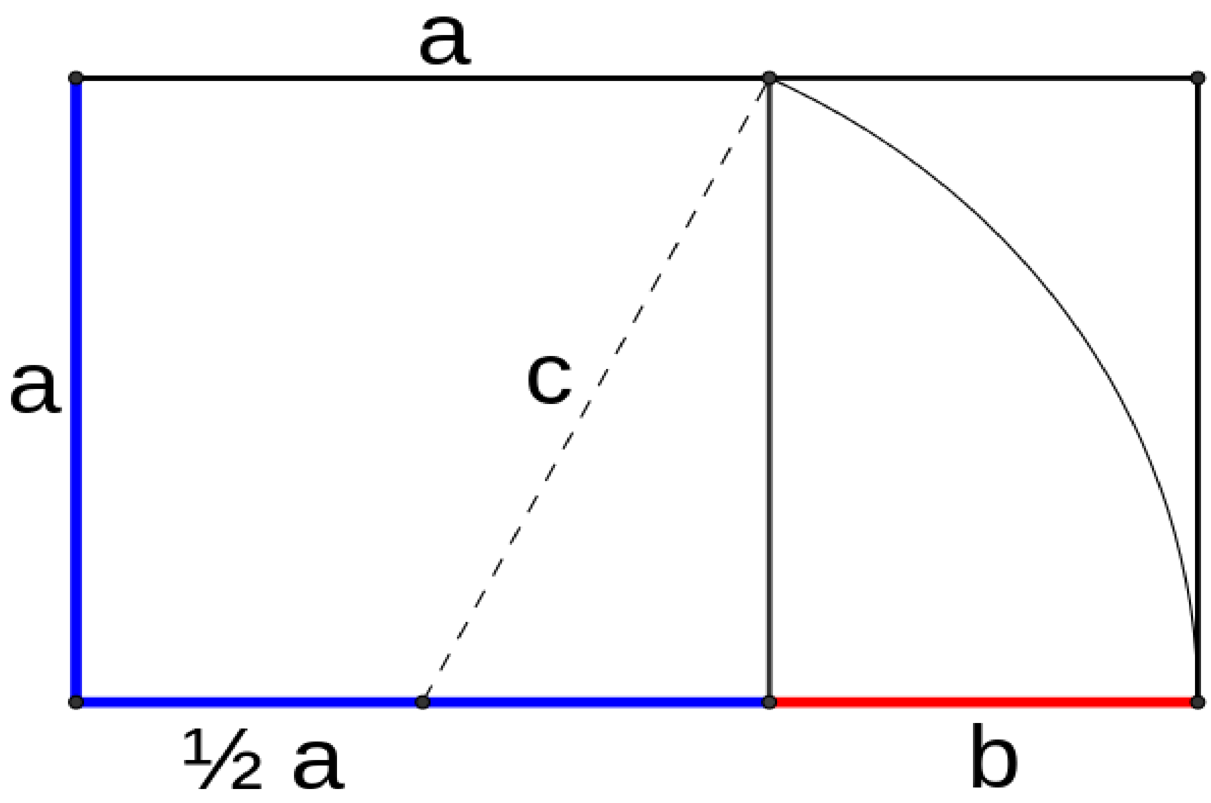
1 2 1.5 1.6666 1.6 1.625 1.615 1.619 1.617 1.618 1.618 …

Rapidly approaches the Golden Ratio

nth Fibonacci number is given by

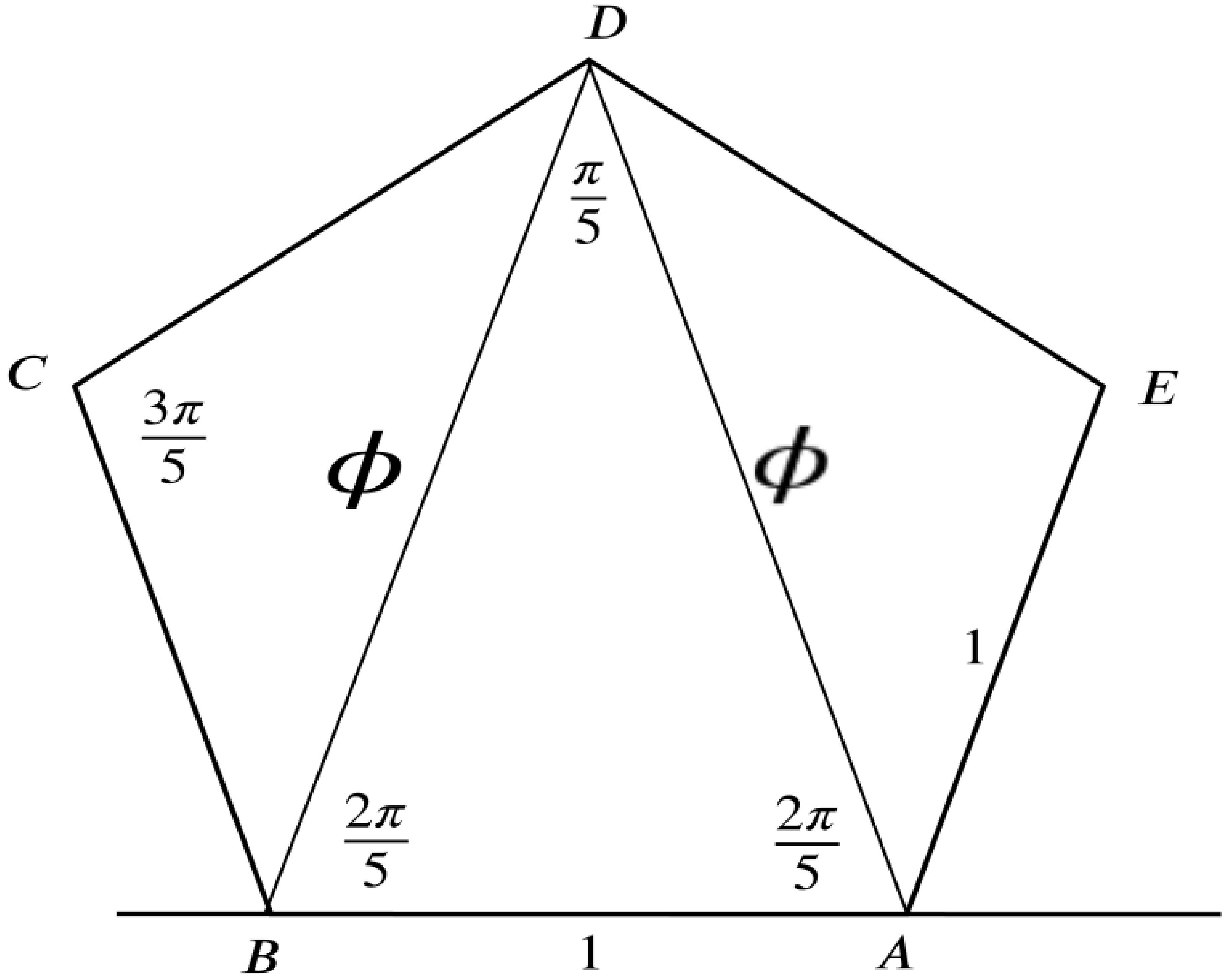
$$x_n = \frac{\phi^n - (1 - \phi)^n}{\sqrt{5}}$$

More Geometry

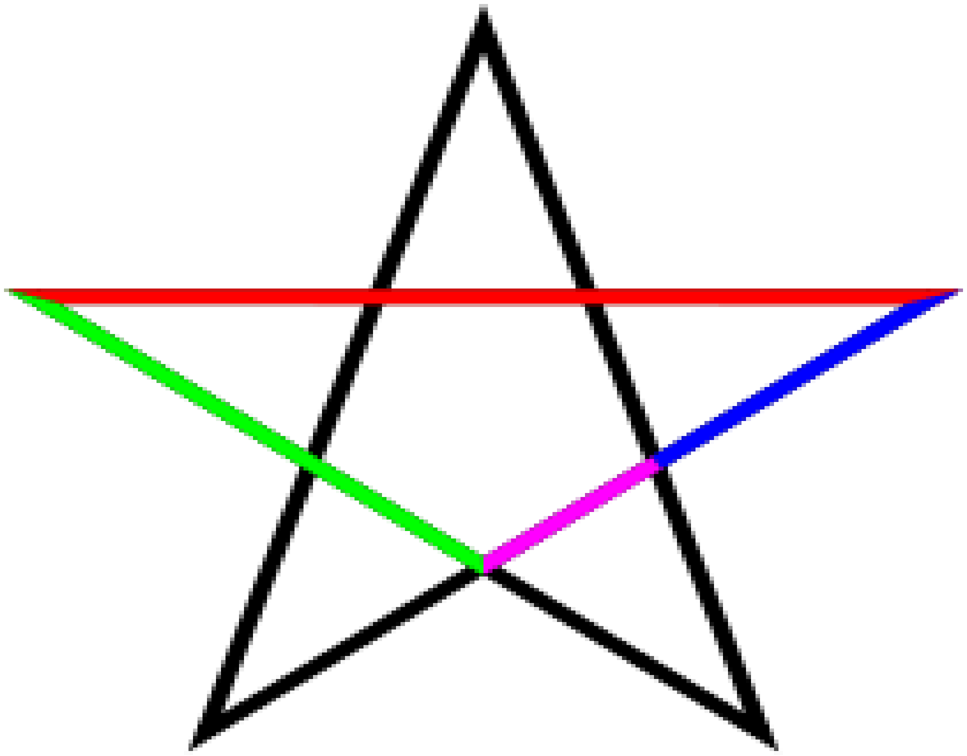


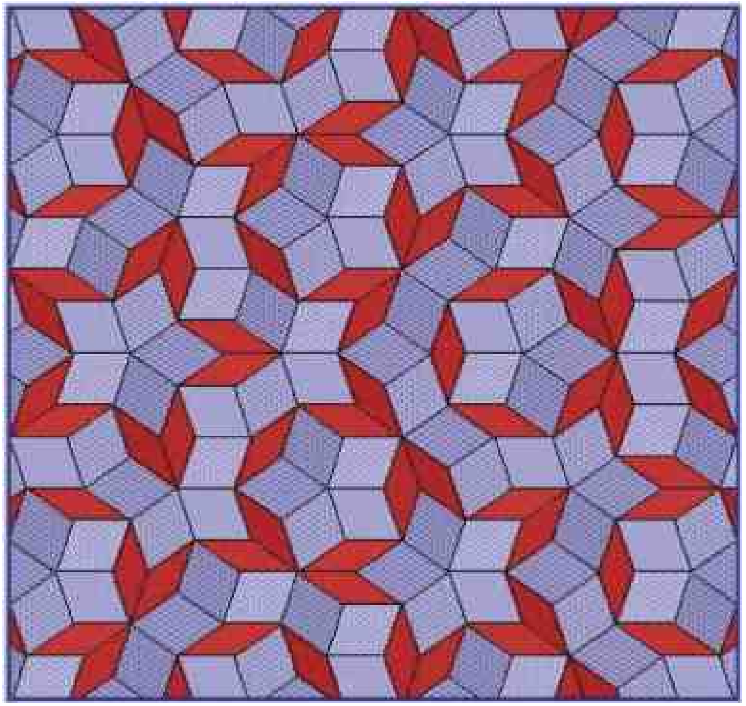
Ruler and compass construction of the Golden Ratio

Link to the Pentagon



The Pentagram and Penrose Tilings





Irrationality of the Golden Ratio

There are no integers m and n such that

$$\phi = \frac{m}{n}$$

So .. Does the the Golden Ratio play a 'central role in mathematics'?

NO!!! This is a myth

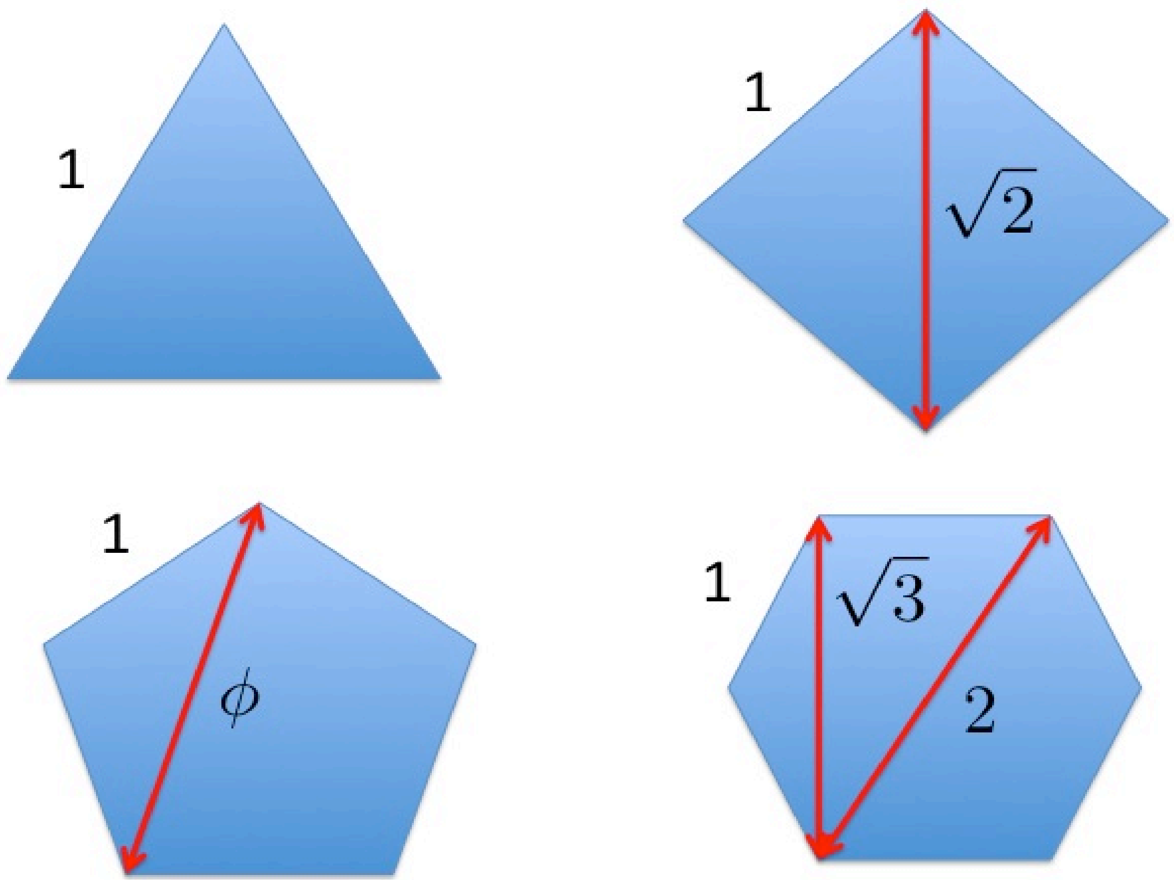
As a number it is only in the Championship League

Premier League

Numbers

Bottom of the Premier league

$$\sqrt{2} = 1.41421356..., \qquad \sqrt{3} = 1.73205081...$$



Top of the Premier league

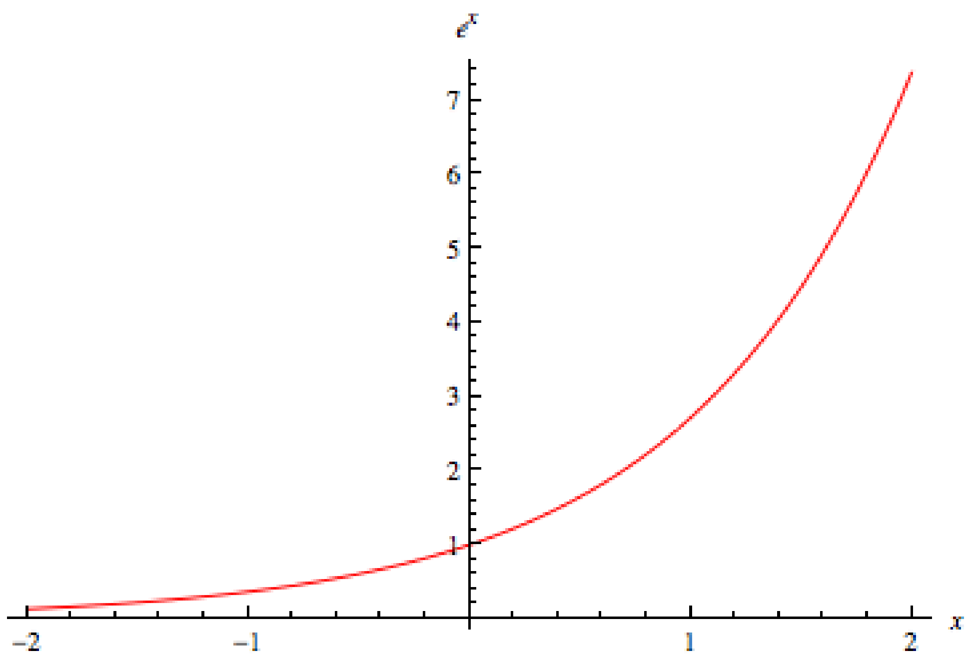
$$\pi = 3.14159265358979323847\ldots$$

4

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots$$

$$e = 2.718281828....$$

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\frac{1}{6!}+\ldots$$



Exponential growth

Practically every formula in science and engineering involves e, or pi or a combination of the two

In my own work I use these two numbers

all the time

I have used the Golden Ratio exactly twice

Other great numbers

0, 1, -1, i

The most important formulae in mathematics

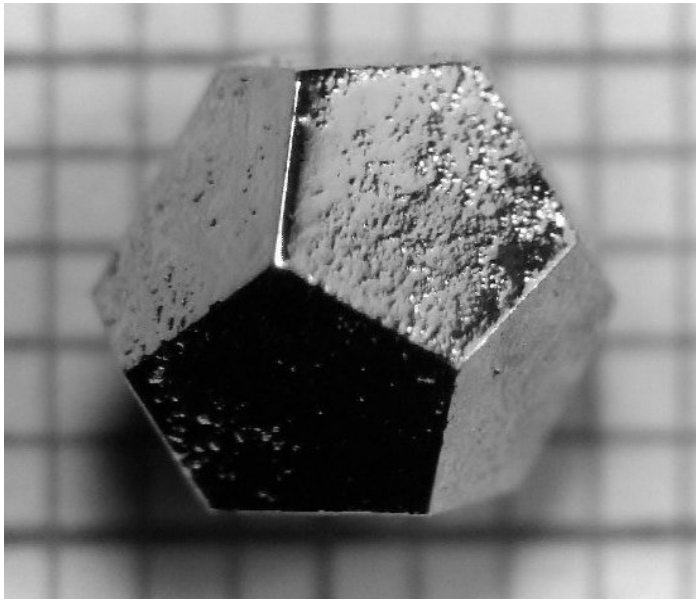
$$e^{i\pi} = -1 \quad \text{or} \quad e^{i\pi} + 1 = 0$$

No sign of the Golden Ratio here!

Links of the Golden Ratio to Nature

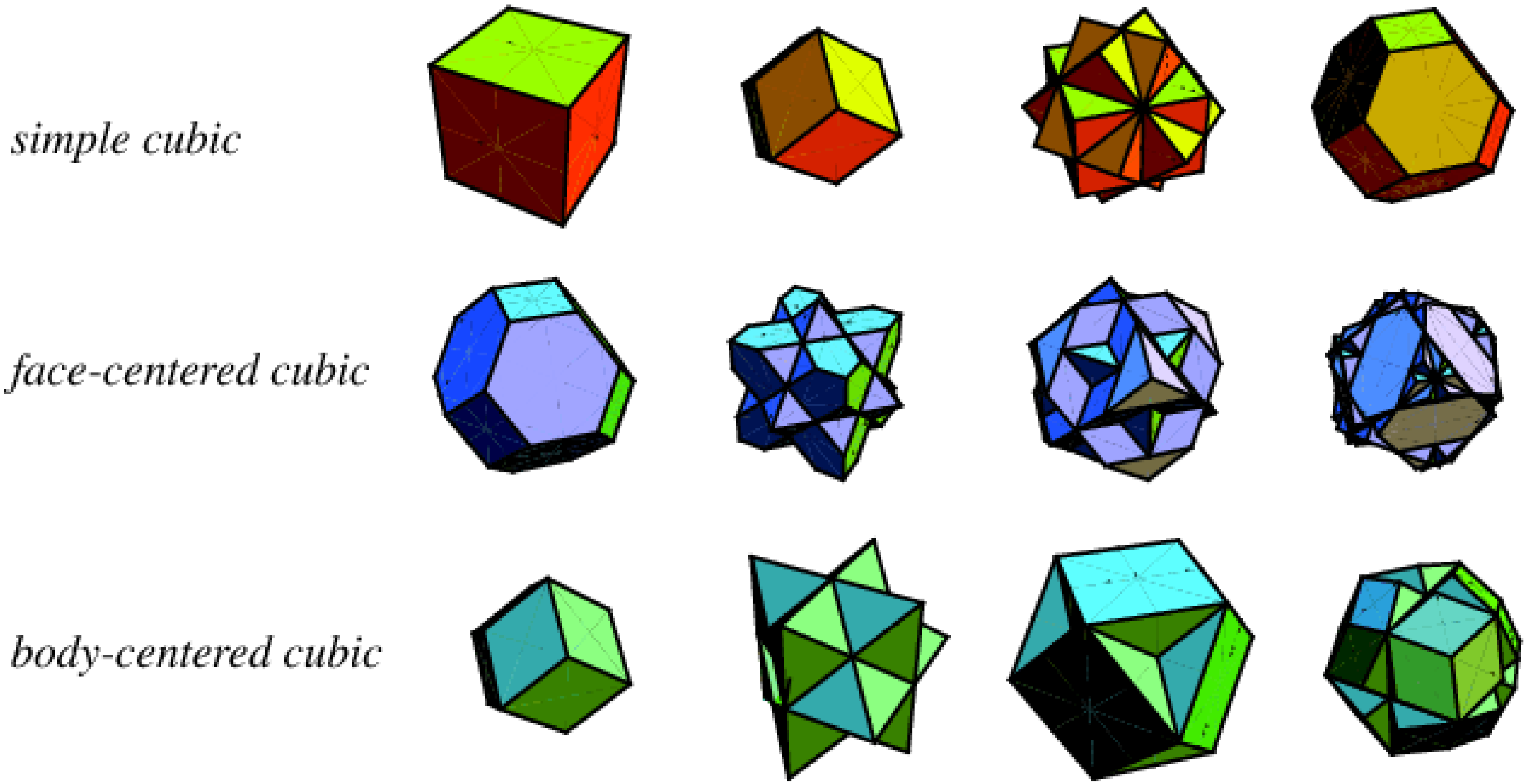
The truth

The Golden Ratio appears in nature when there is five-fold symmetry



Eg. Quasi crystals

However these crystals are rather uncommon compared to cubes and hexagons which involve the square roots of 2 and 3



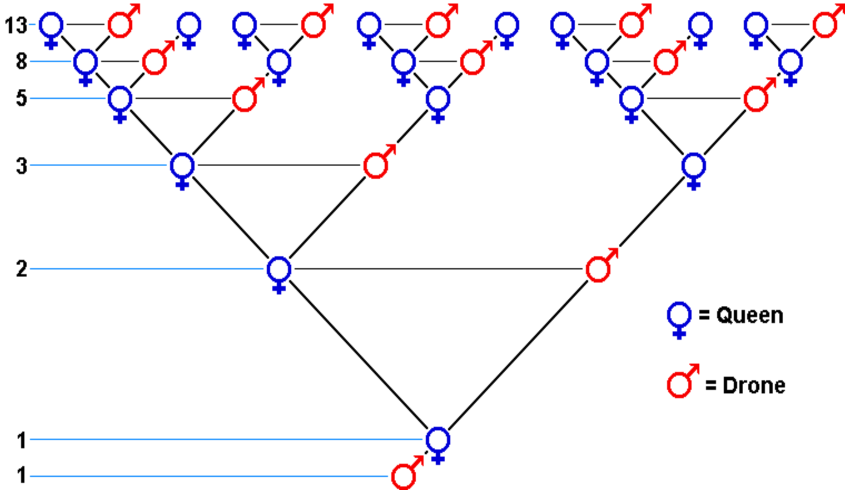
The Golden Ratio is linked to the Fibonacci sequence

$$x_{n+1} = x_n + x_{n-1}$$

This arises naturally in studies of population growth

and also the way that objects pack together

Spirals in a sunflower



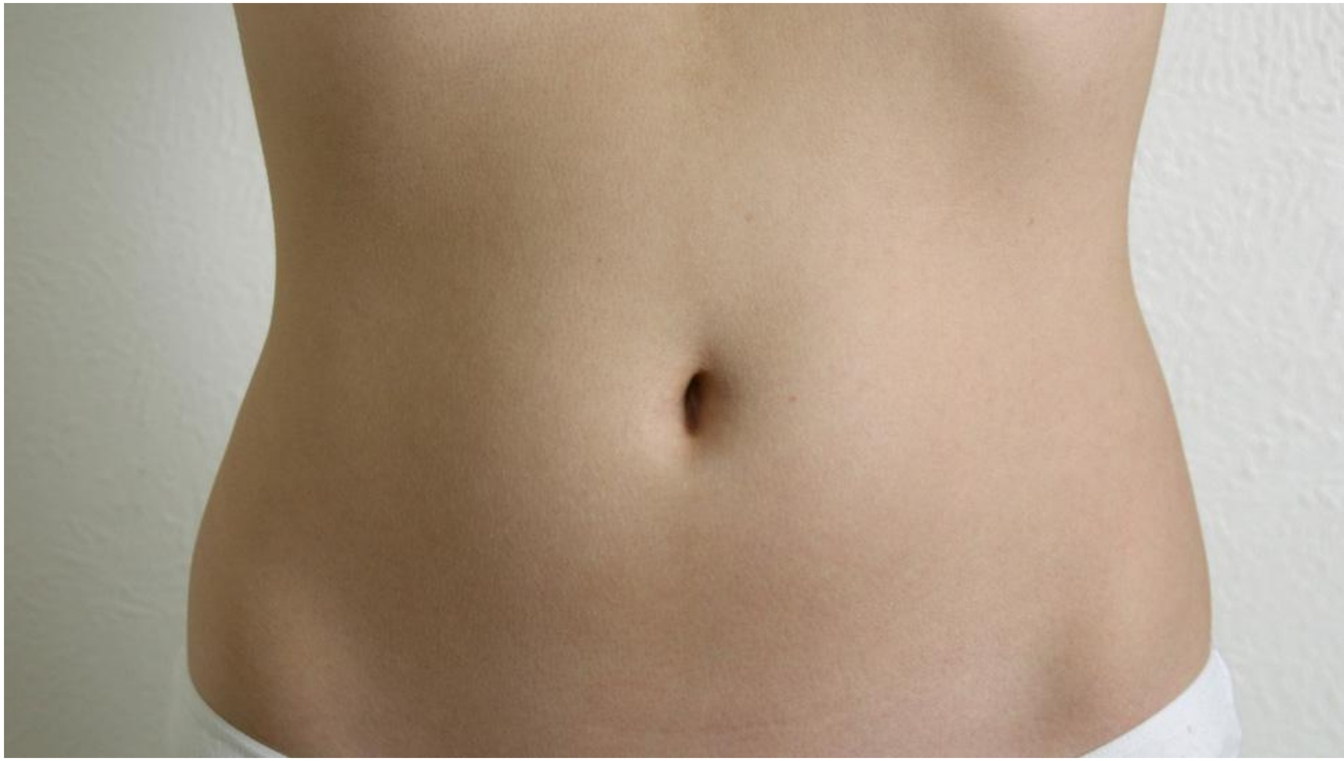
Drone bees in a bee hive

The myth

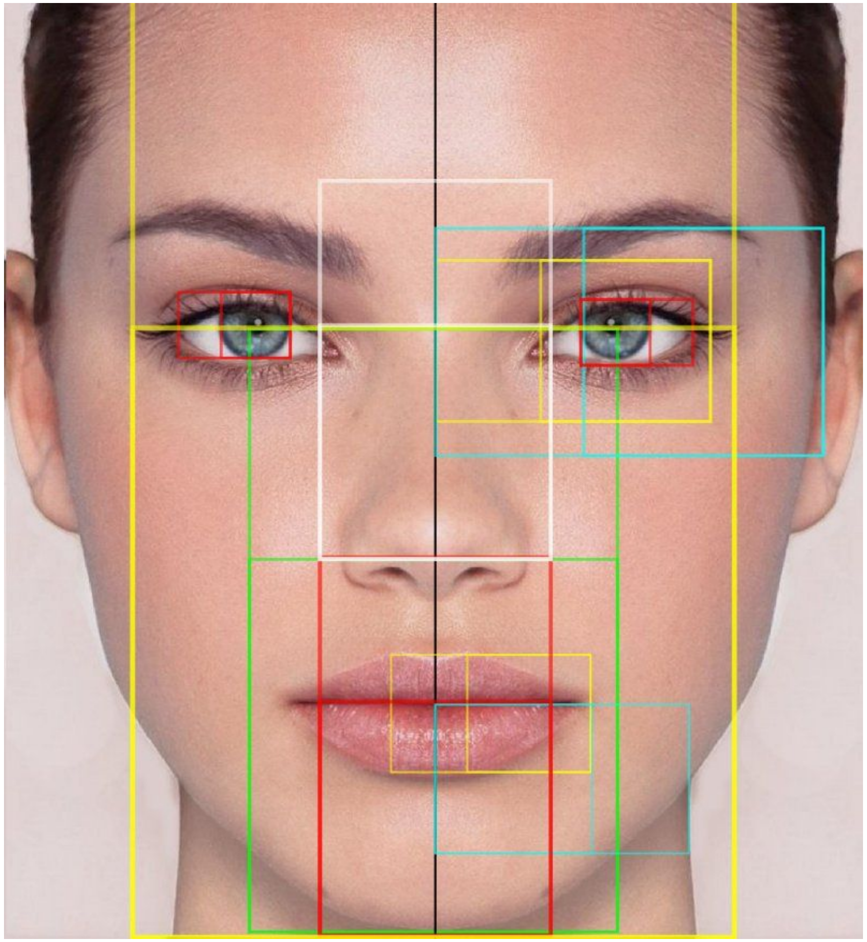
Much much more is claimed of the Golden Ratio

Supposed to be at the heart of many of the proportions in the human body.

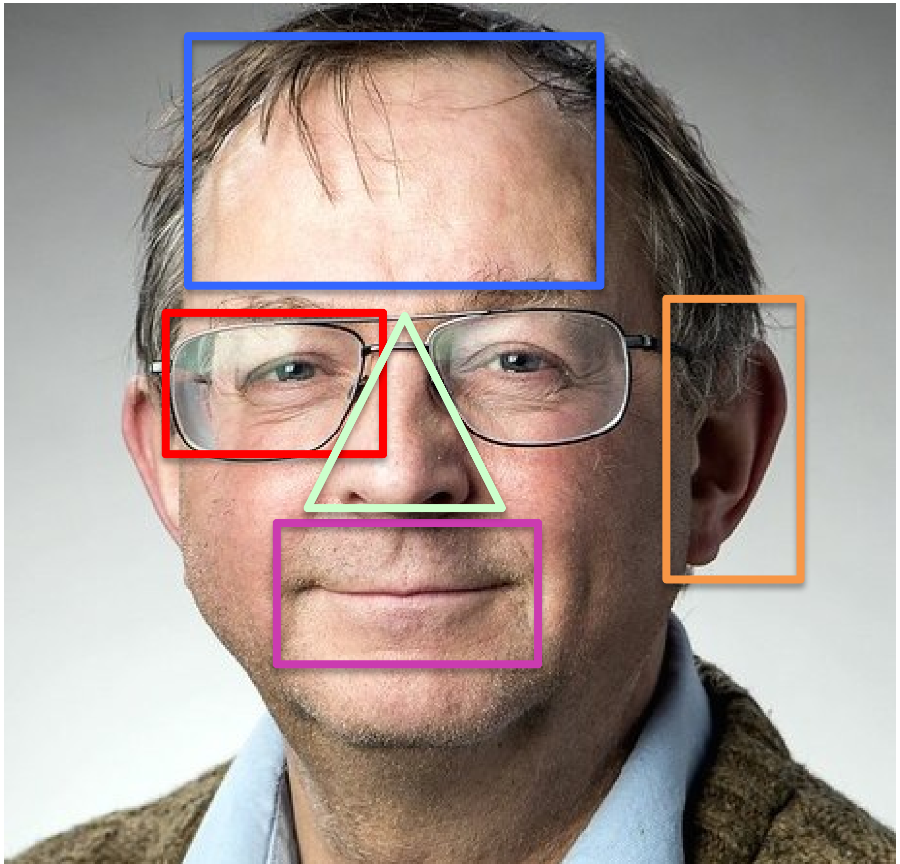
Eg. ratio of the height of the navel to the height of the body



Just about every proportion of the perfect human face

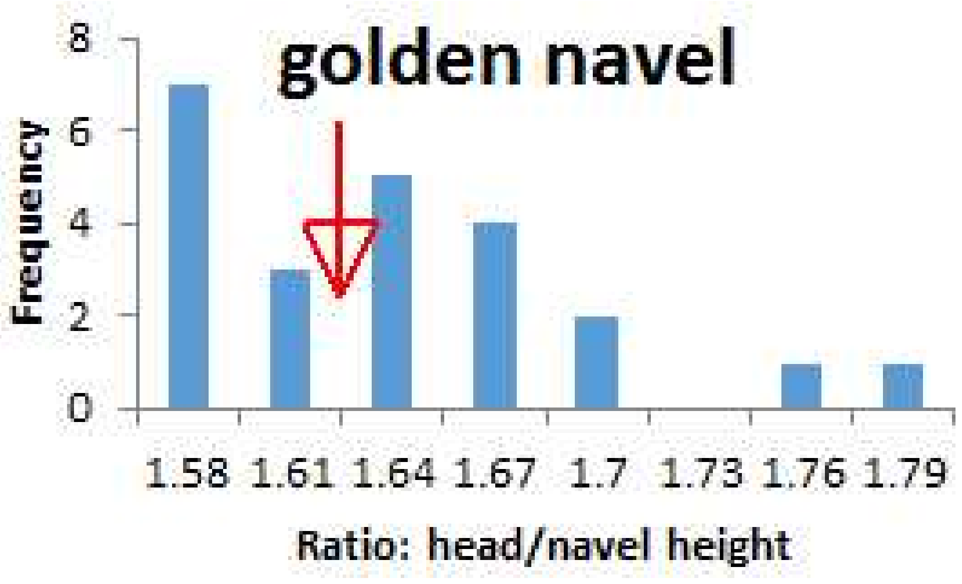


However, none of this is true, not even remotely !!!



The human body has lots of ratios between 1 and 2

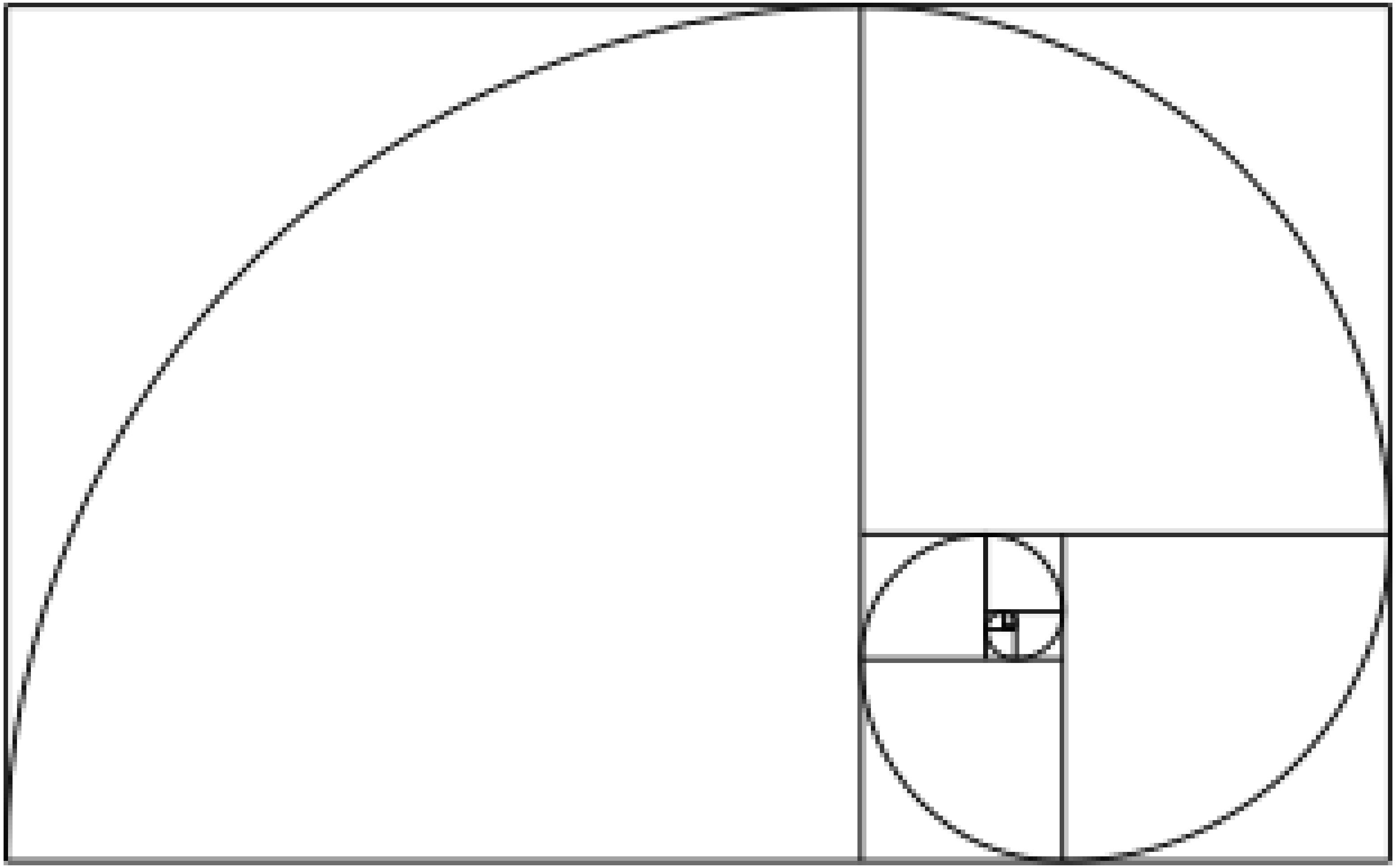
Some by chance are ‘close’ to the Golden Ratio



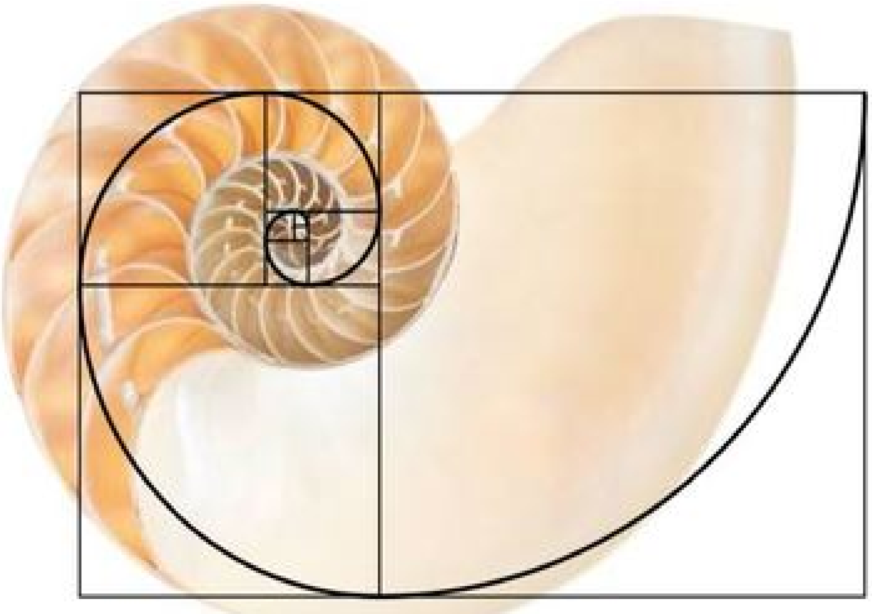
Many are not

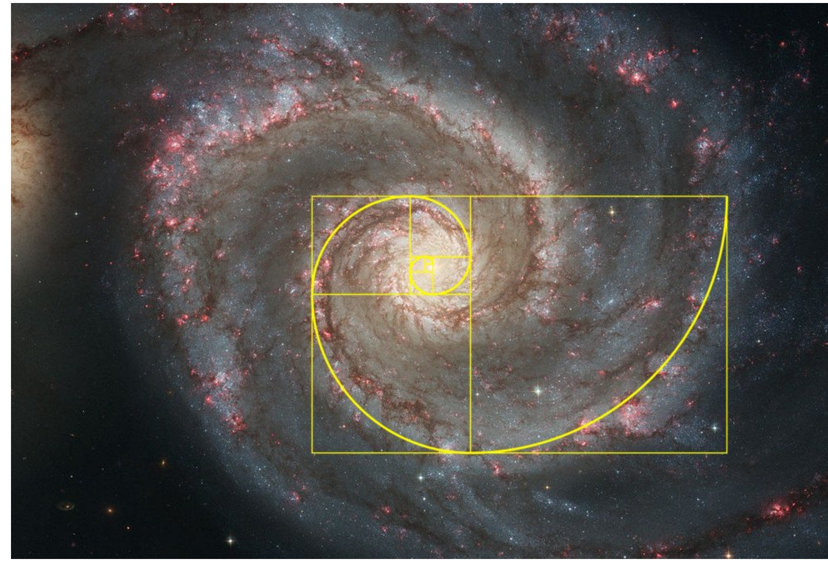
Tendency of the human brain to look for patterns

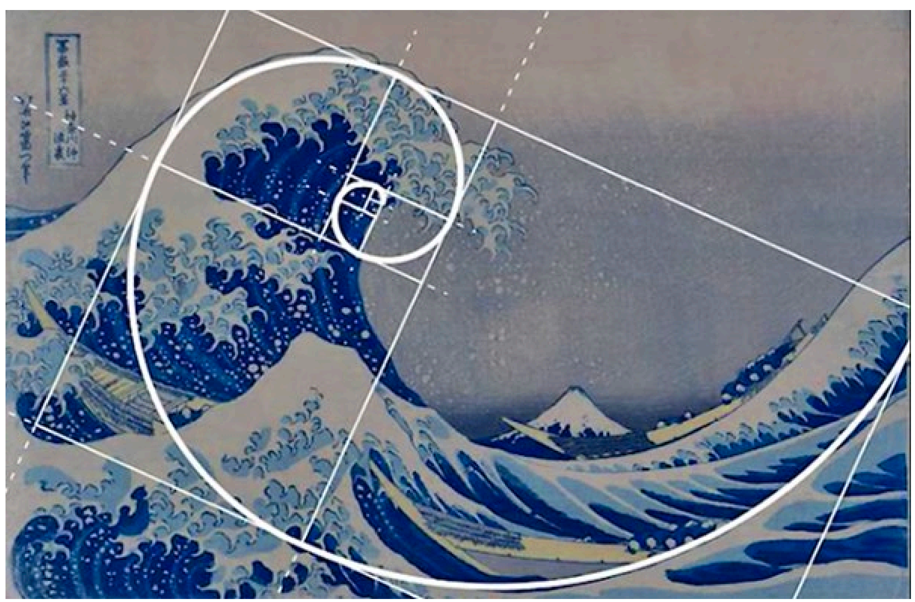
Spirals, Golden and otherwise



Golden spiral …. It's not a spiral!







Logarithmic Spiral

$$r = a\, e^{b\,\theta}$$

Very common in nature

Because of … Self-similarity

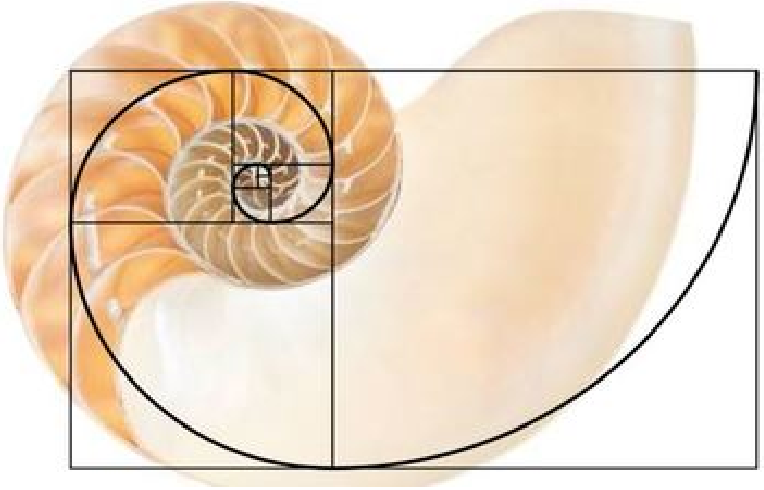
If you rotate the spiral by any fixed angle then you get a spiral which is a rescaling of the original a and b can be any value!!

Golden Spiral

$$b = \ln(\phi)/(\pi/2) = 0.3063489..$$

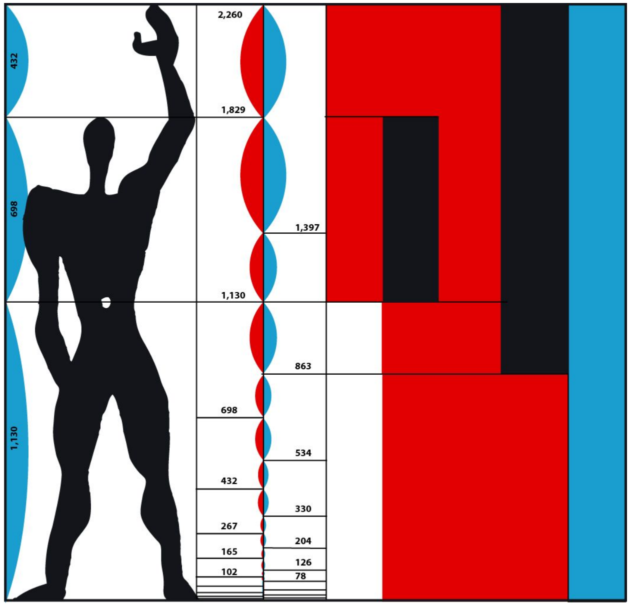
Nautilus:

$$b = 0.18$$



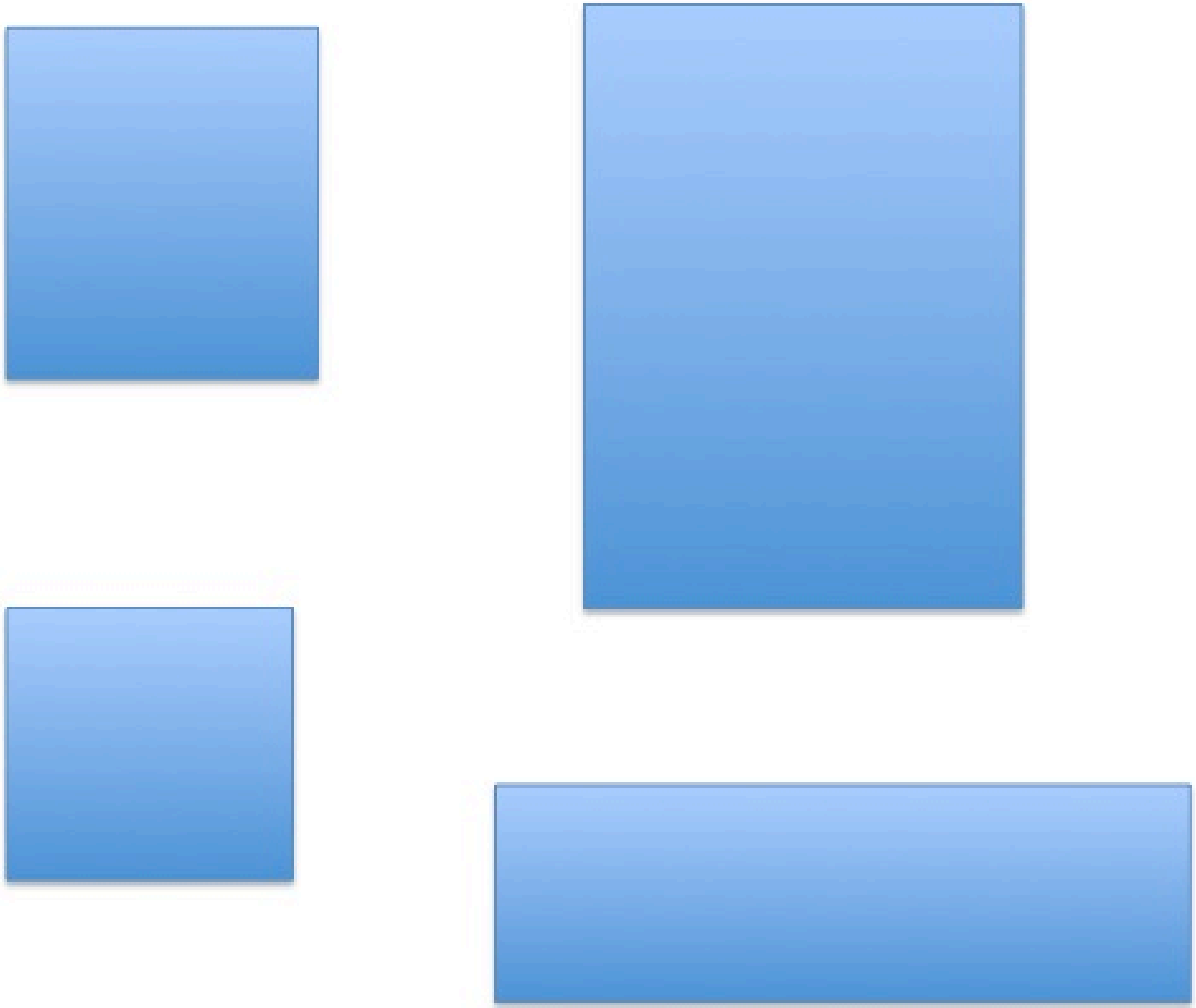
The Golden Ratio in Art

The truth: Some artists eg. Le Corbusier have consciously used the Golden Ratio

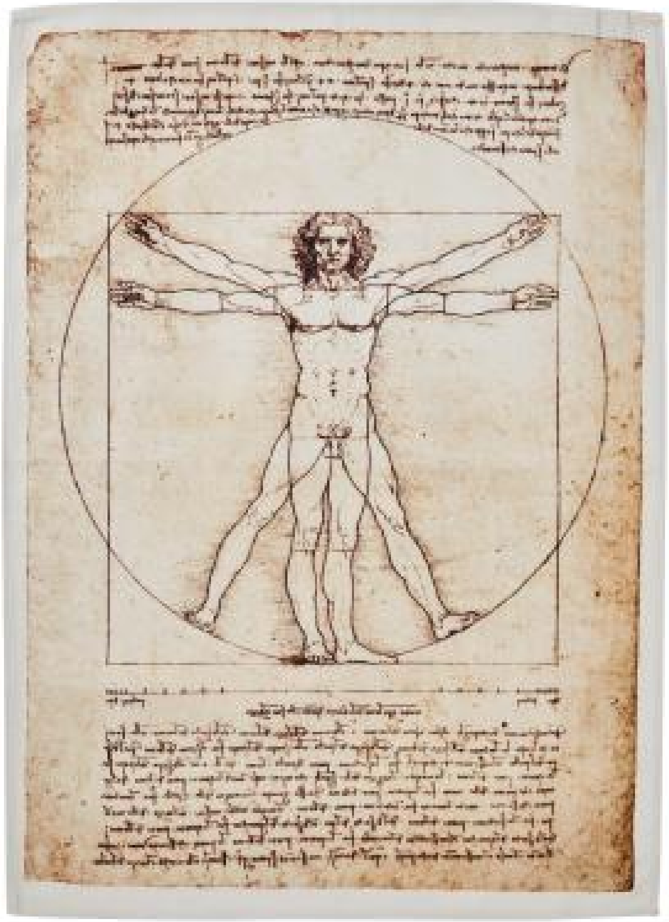


The myth: The Golden Rectangle is supposed to be the most aesthetically pleasing rectangle

This has been tested. There is no evidence for it



Claimed da Vinci used the Golden Ratio in his art



No direct evidence of this!

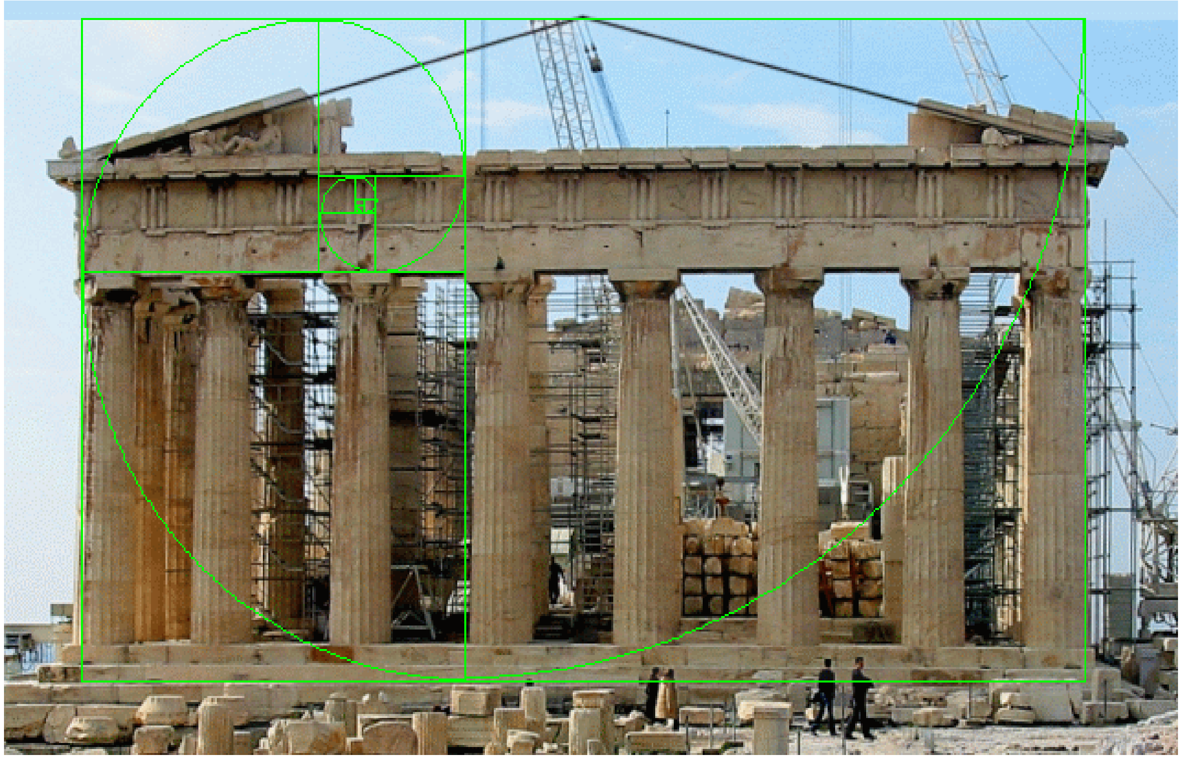
da Vinci only mentioned whole number ratios

Famous example: *Vitruvian Man*

But its proportions do not match the Golden Ratio

Examples of finding the Golden Ratio in his pictures are in the same class as finding the ratio in the face

The Parthenon



No evidence of the use of the ratio in Greek scholarship

Idea the Parthenon has proportions given by the Golden Ratio only dates back to the 1850s

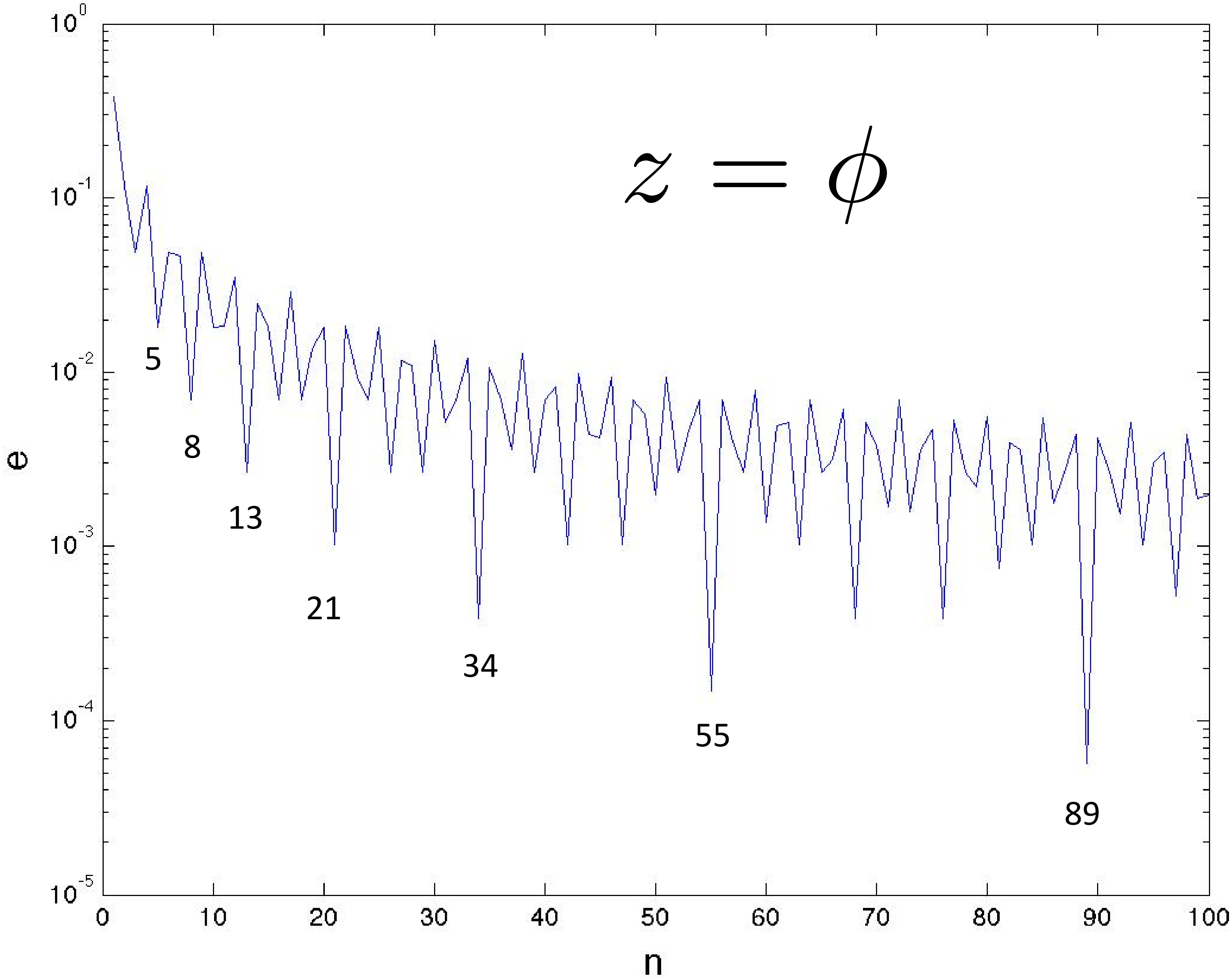
The real truth is much better than the myth

The Golden Ratio is the most irrational number

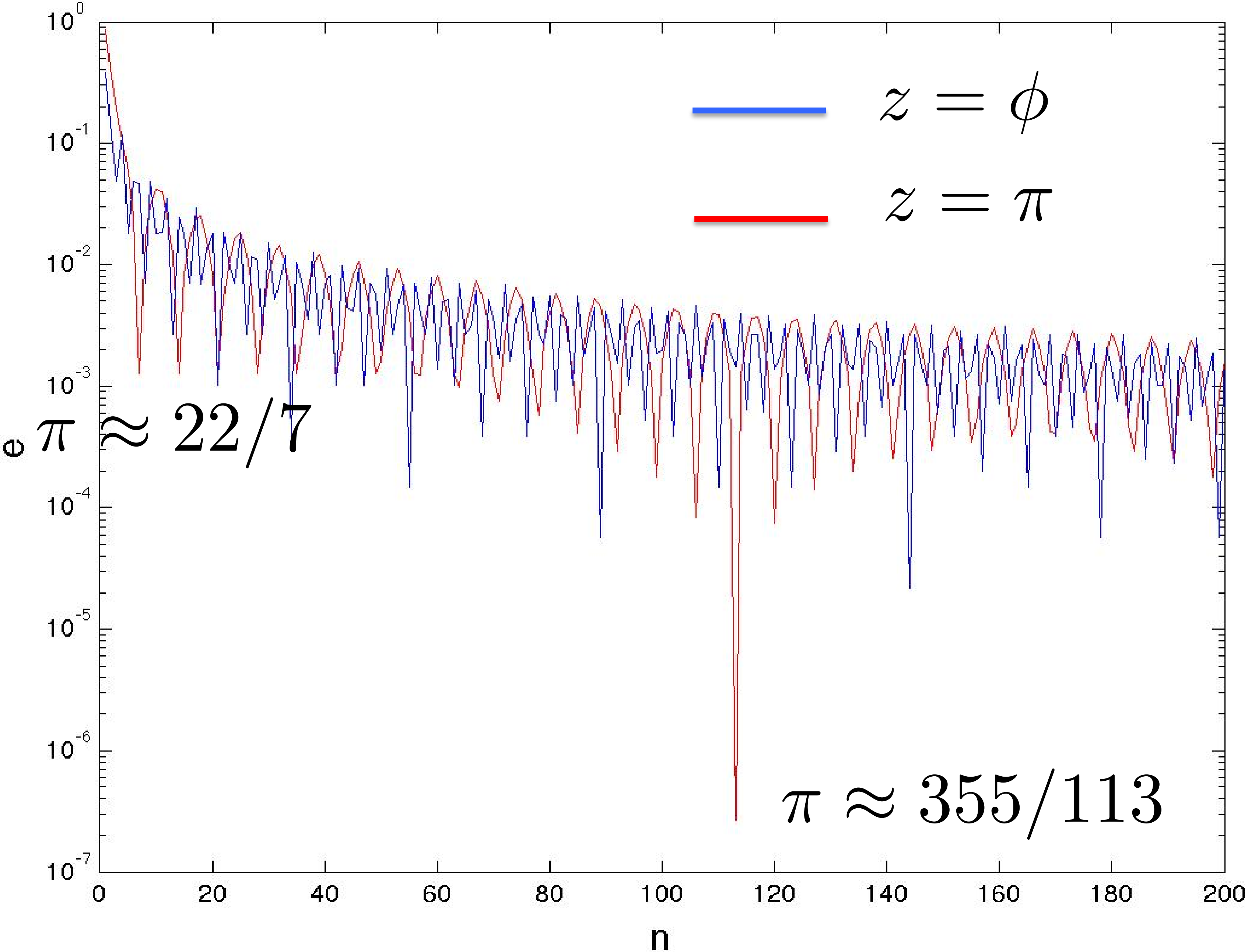
z any number m/n a fraction

$$e = |z - m/n|$$

error of approximating z by a fraction



$z = \phi$
e
n
5
8
13
21
34
55
89
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10
20
30
40
50
60
70
80
90
100



$z = \phi$
$z = \pi$
$\pi \approx 22/7$
$\pi \approx 355/113$
e
n
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
20
40
60
80
100
120
140
160
180
200

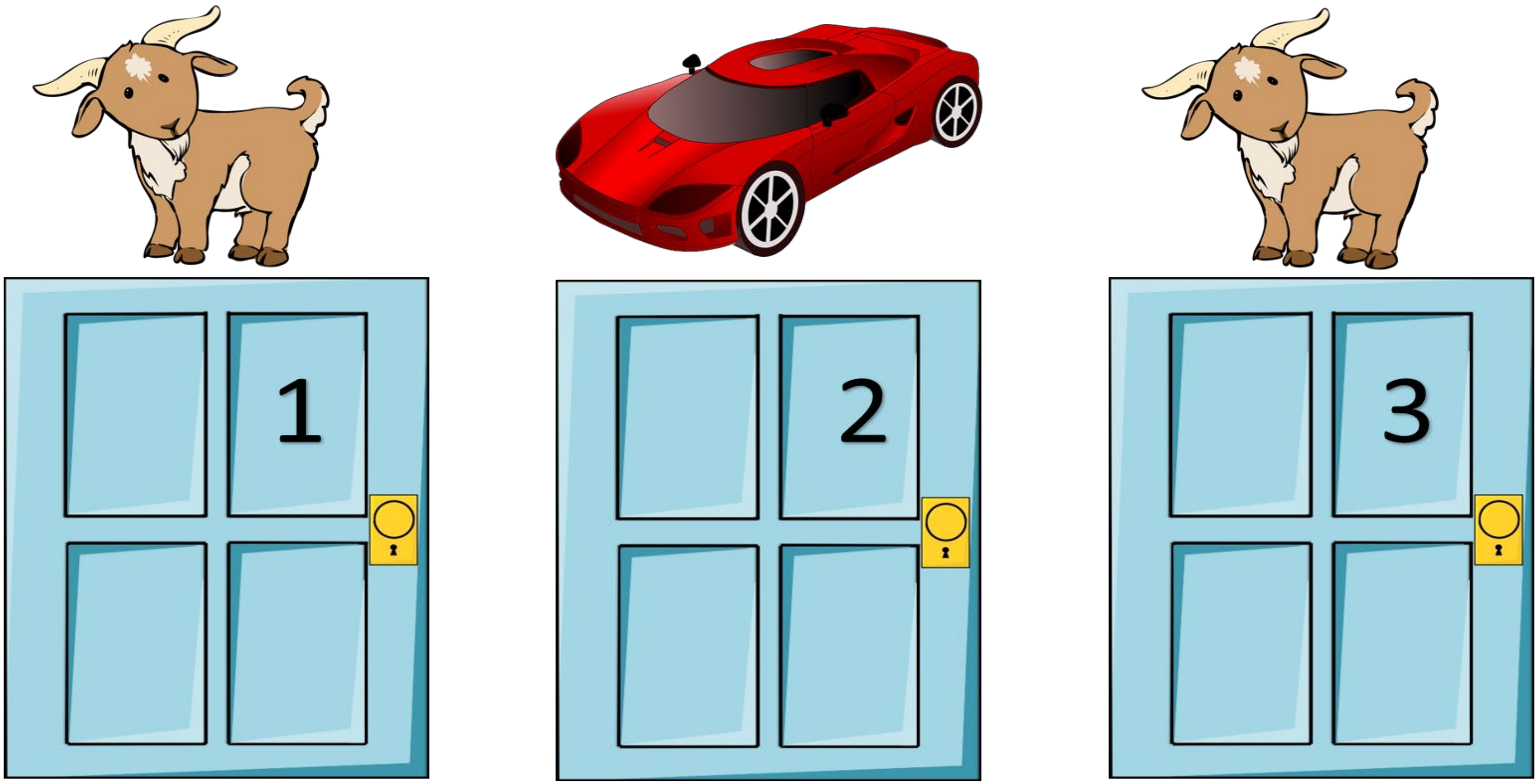
$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \dots}}}$$

Hard to approximate

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \dots}}}}$$

Easy to approximate

The Monty Hall Problem



Contestants on the game show are shown three shut doors

Behind one is a car

Behind the other two is a goat

Asked to choose a door and to tell the host

The host opens a different door to reveal a goat

Given a choice

Stay with the chosen door

or swap

The door they choose is opened

Should you change your door or not?

Often quoted answer is YES!

But this isn't always correct

Answer depends completely on the knowledge of the host and contestant

Case 1: Knowledgeable host, ignorant contestant

Chance contestant chooses a car is 1/3

Knowledgable host will always reveal the goat so no new information

Chance the other door is a car is still 2/3

Double your chances by changing doors

Case 2: Ignorant host, any contestant

Contestant chooses a door

Host shuts their eyes and opens a door. It's a goat

Changes the information in the problem

Bayes' theorem says that the chance of the contestant having a car is now ½

No reason to change

Case 3: Knowledgeable host, knowledgeable contestant

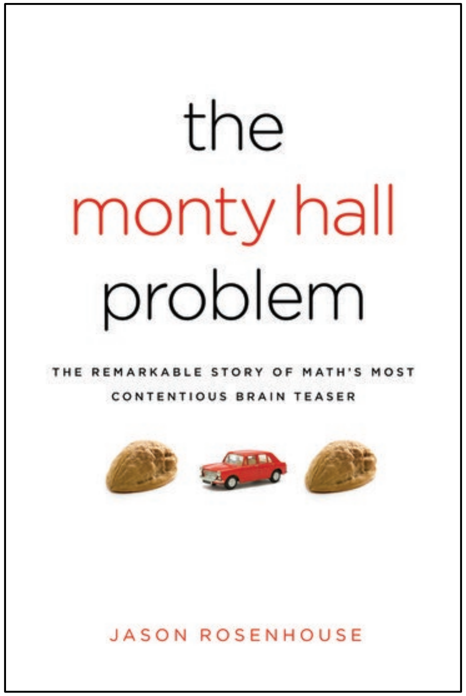
A. Contestant chooses a car. Host reveals goat. Contestant changes their door. They get a goat!

B. Contestant chooses a goat. Host asks them to choose another door, and then choose whether to swap doors. They swap and get a car with probability ½

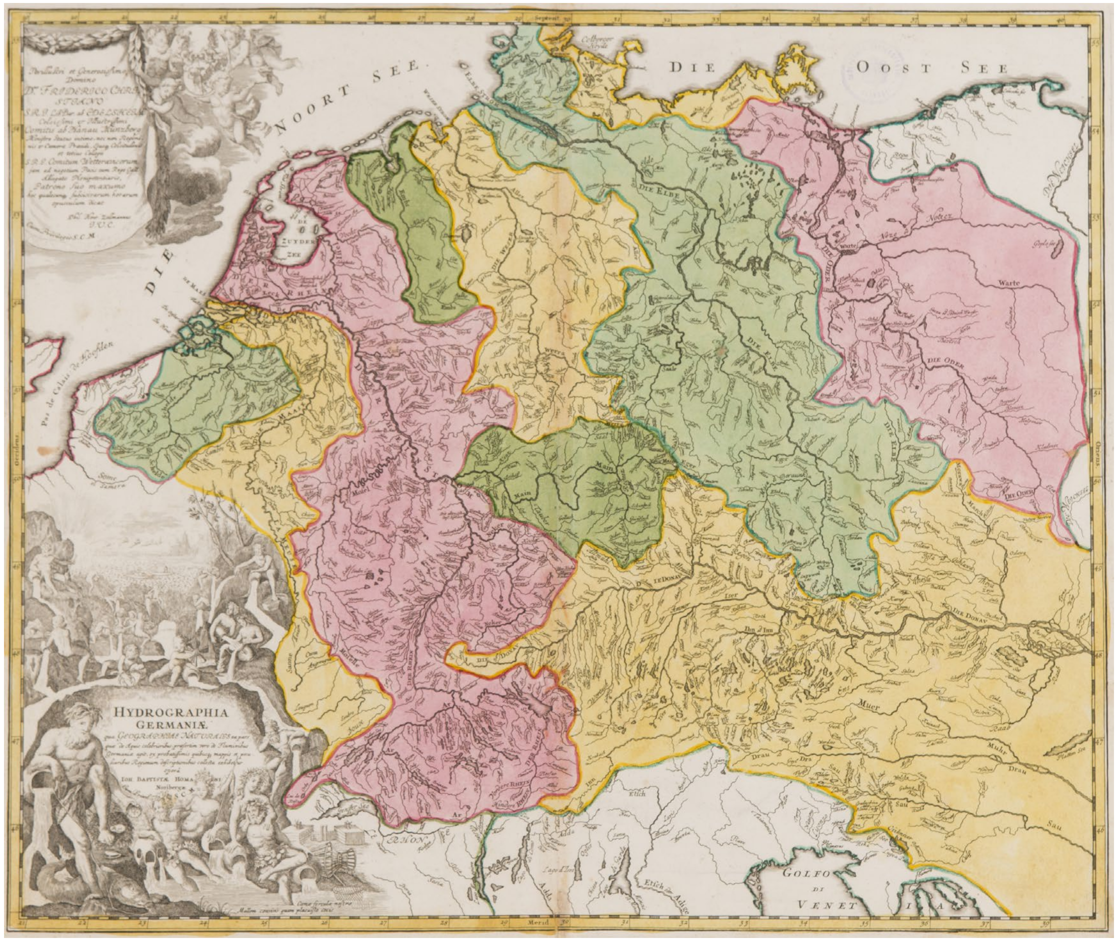
Overall they only get a car with probability 1/3

A little knowledge is a dangerous thing

Rob Eastaway



The Four Colour Theorem

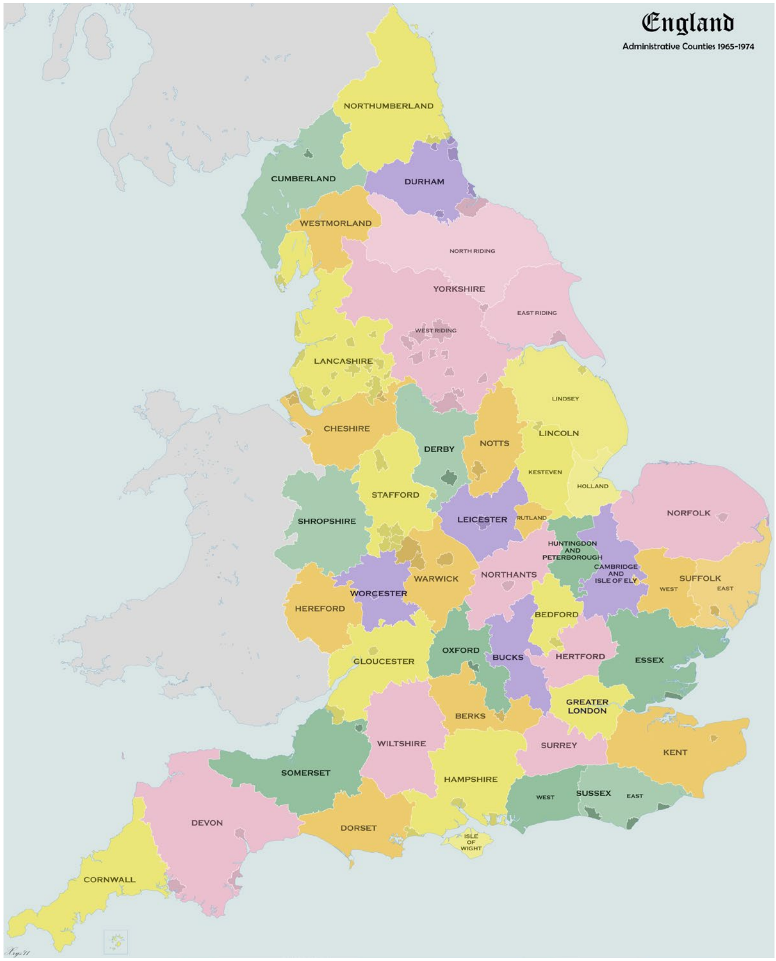


Want to colour a map as cheaply as possible

Rules

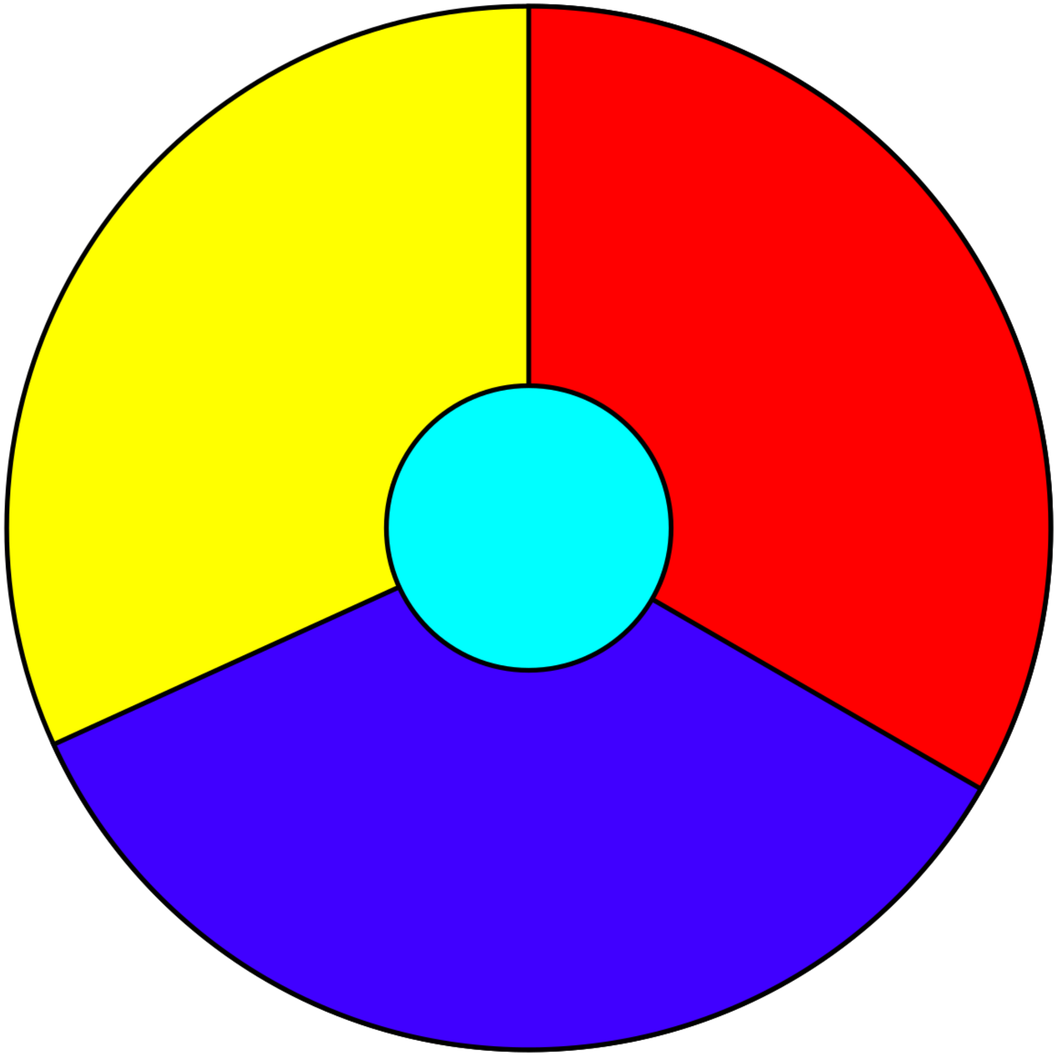
1. Each country must have
 one colour

2. Two countries which
 have a common border
must have different colours

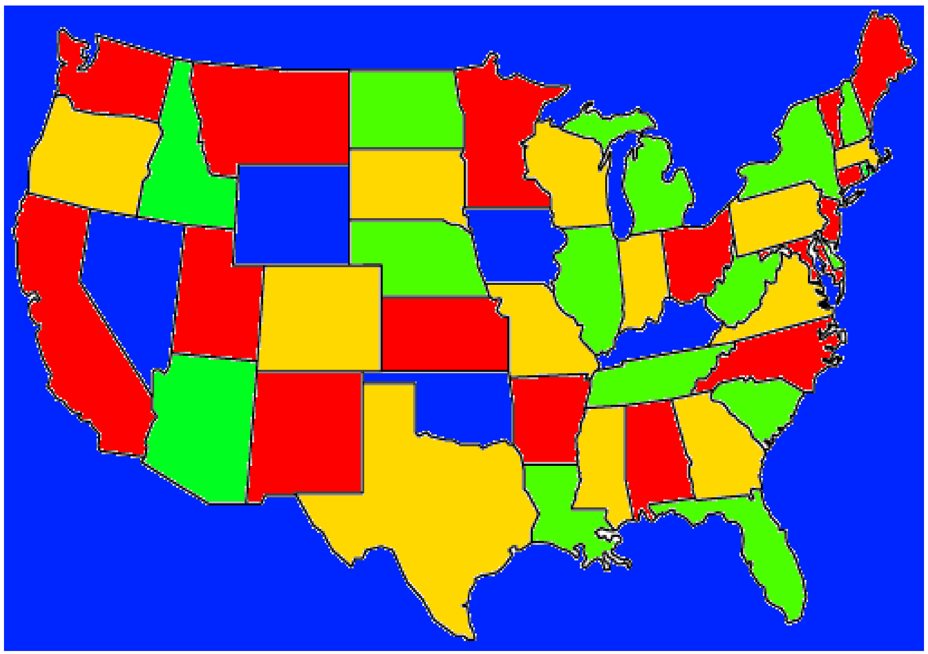


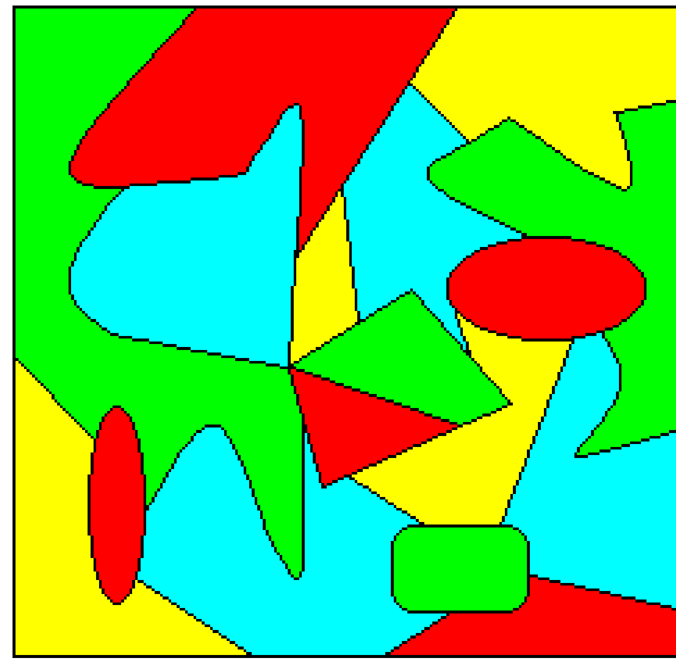
Q. What is the smallest number of colours needed?

Must have at least four colours



Found 'empirically' that only four colours seemed to be needed





Conjecture proposed in 1852 by Francis Guthrie, who was trying to colour the map of counties of England

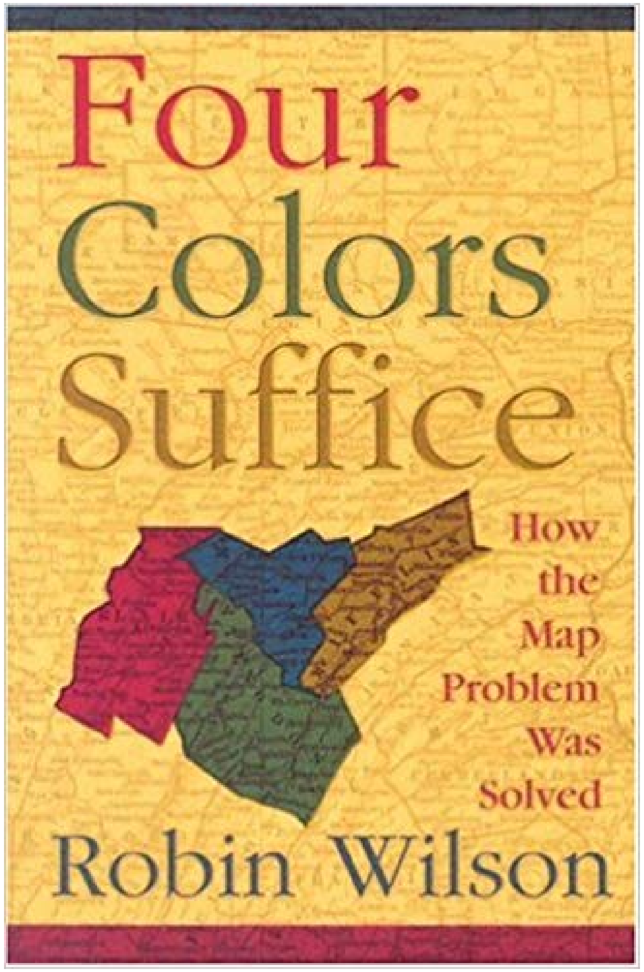
‘Proof’ in 1879 by Kempe

Shown to be wrong. But became the ‘five colour theorem’

Resisted proof for 100 years

1976 the four colour theorem was proved by Kenneth Appel and Wolfgang Haken

Proof was ‘by computer’ and was Very controversial



Now a very important result in communications technology

The Truth

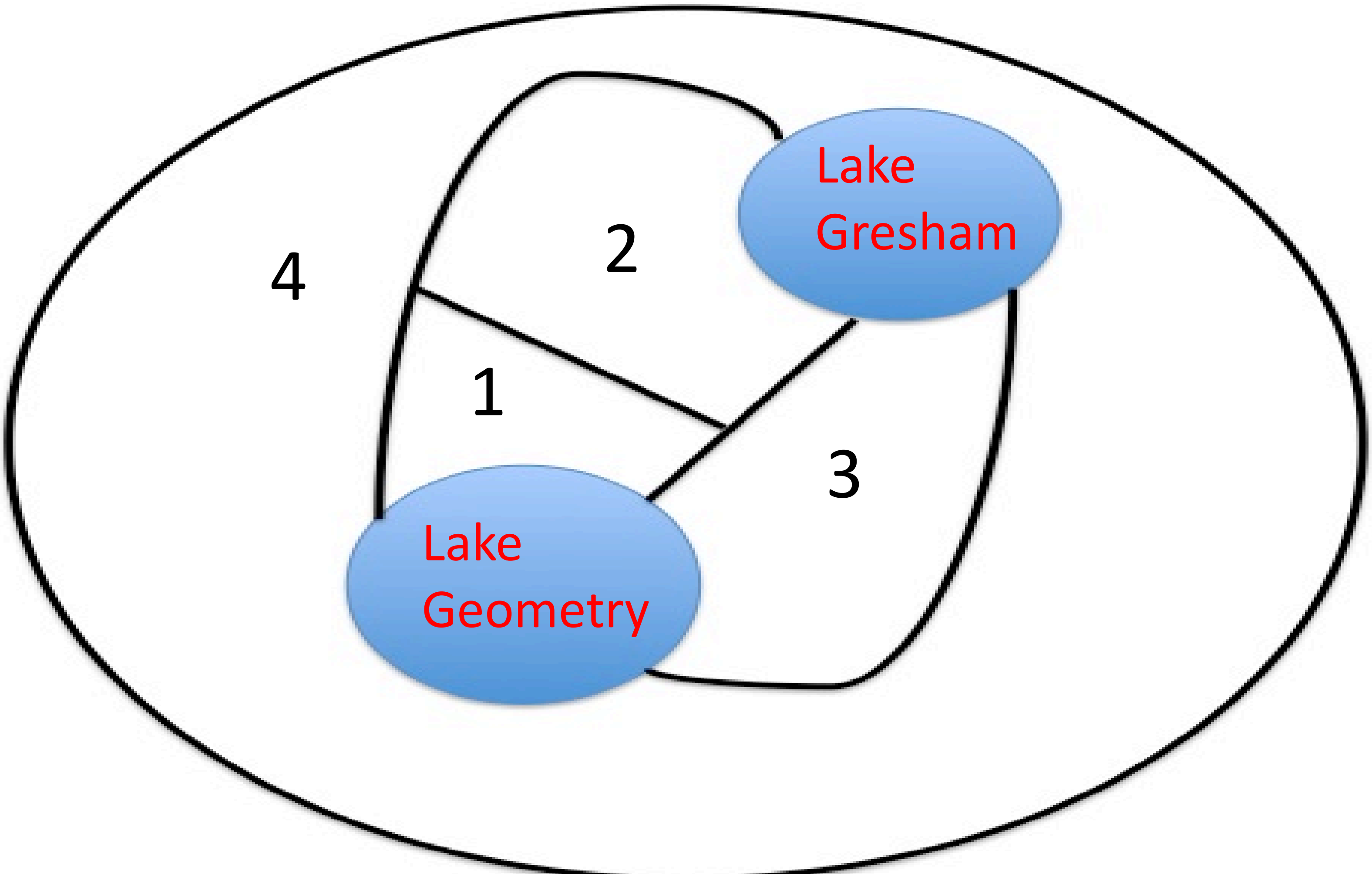
The four colour theorem applies to all non contiguous planar graphs

The Myth

It works for maps

It doesn't!

A map which cannot be coloured with four colours

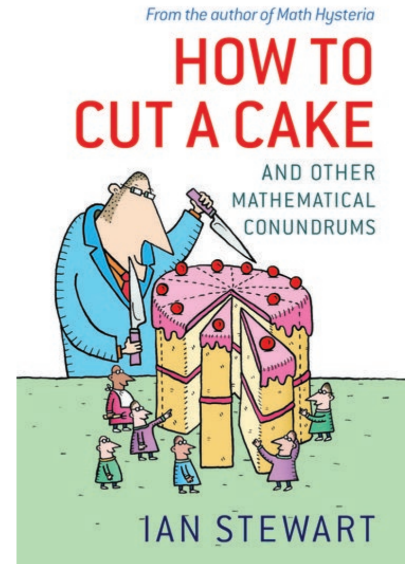


Problem: The sea is always blue

Empires can be red

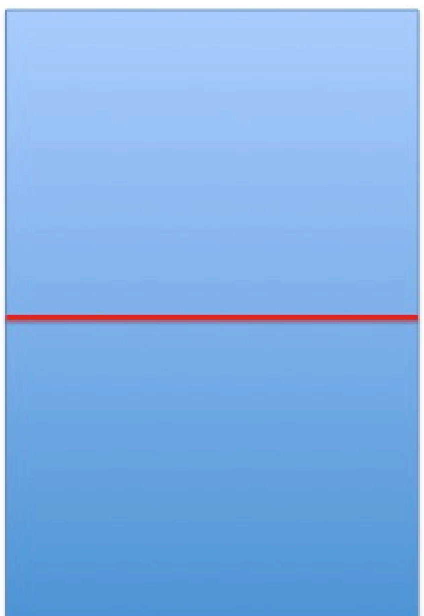
Maps like this often need more than four colours

Cutting a cake

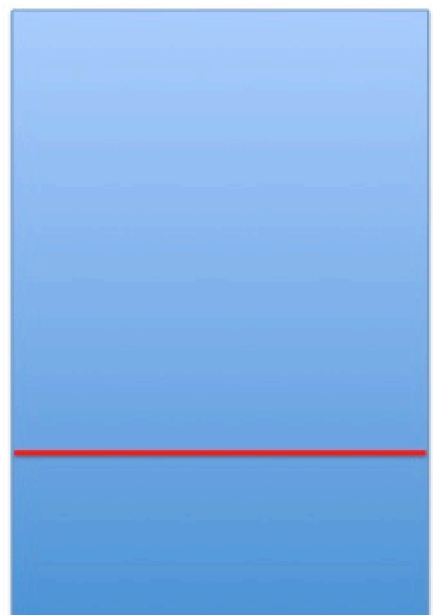


How do you cut a cake fairly?

Fair cut



Unfair cut



The Myth: *The I cut, you choose, method*

One party divides up the cake
The second party then has the first choice

Reasoning: it is clearly in the interests
of the first party to cut the cake as fairly as possible

Then, no matter how the second party choses, the remaining piece will be as close to ½ of the original as it is possible to get

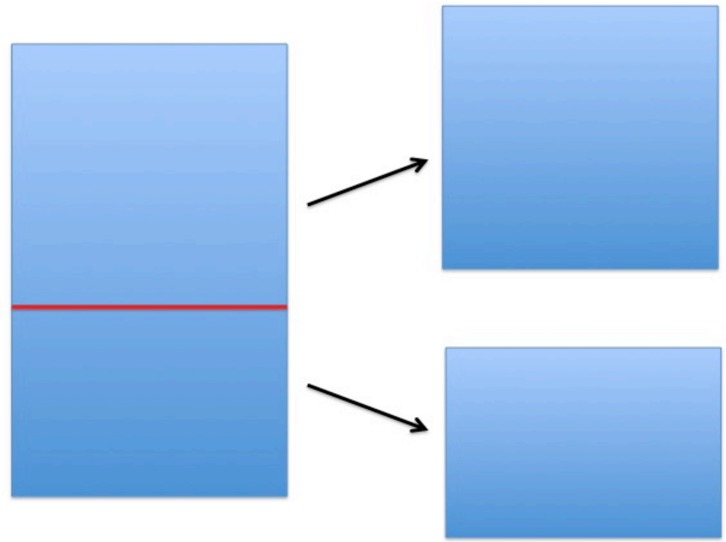
Problem:

This method gives an overwhelming advantage to the person who choses first

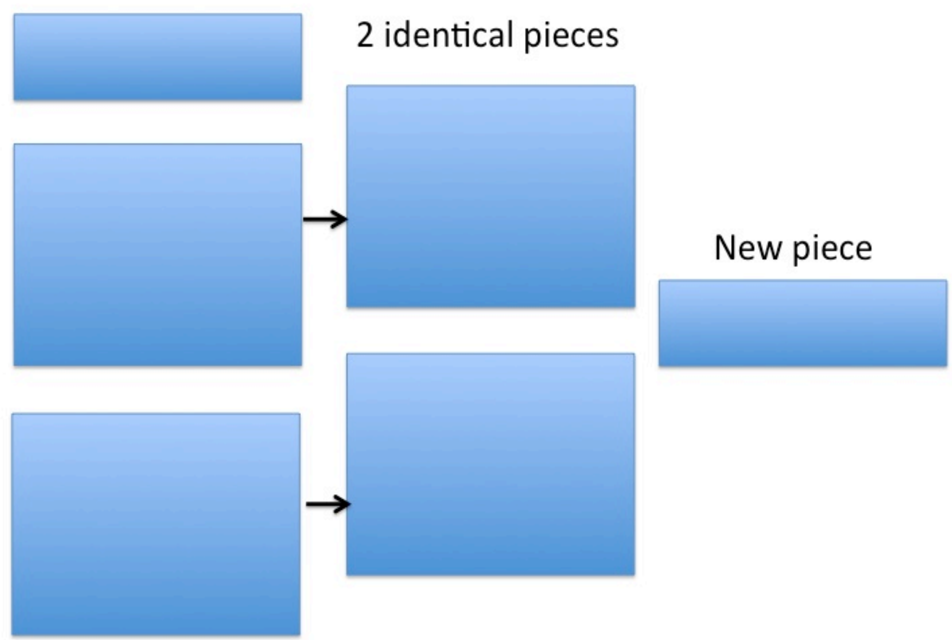
A much better method is to use iteration

This is how a computer would do it

First Cut



Second Cut



Continue with the new piece till only the crumbs are left

A mathematician named Hall,

Once went to a fancy dress ball,

They thought they would risk it,

And go as a biscuit,

But a dog ate them up, crumbs and all.